Math 351 Homework, Due Friday, November 21.

1. Determine whether the set Z of vectors is linearly independent

$$Z = \{[1,\ 2,\ 2,\ 1],\quad [-1,\ 2,\ 1,\ 1],\quad [4,\ 2,\ 0,\ 1]\}\,.$$

2. Find a basis and the dimension for the span of the given vectors (i.e. find the largest linearily independent subset of vectors)

   (a) $[1,\ 2,\ 2],\quad [3,\ 2,\ 1],\quad [11,\ 10,\ 7],\quad [7,\ 6,\ 4]$

   (b) $[0,\ 1,\ -2,\ 1],\ [3,\ 1,\ -1,\ 0],\ [2,\ 1,\ 5,\ 1]$

3. Show whether the following set is a basis.

   (a)
$$[2,\ 0,\ 1],\quad [0,\ -1,\ 2]\quad [1,\ -1,\ 0]\quad \text{for}\quad R^3$$

   (b)
$$[1,\ 0,\ 0,\ 1],\quad [0,\ 1,\ 0,\ 0]\quad [1,\ 1,\ 1,\ 1]\quad [0,\ 1,\ 1,\ 1]\quad \text{for}\quad R^4.$$

4. Given the 3×5 matrix
$$A = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 1 & 1 \\ 2 & 3 & 1 & 1 & -1 \end{pmatrix}$$
   Find a basis (linearly independent vectors) for the column space and the row space of A. What is Rank(A)?

5. Find all possible values of rank(A) a α varies, where $A = \begin{pmatrix} 1 & 2 & \alpha \\ -2 & 4\alpha & 2 \\ \alpha & -2 & 1 \end{pmatrix}$

6. Find the inverse A^{-1} of the following matrices

$$(a)\ A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 0 & 3 & -1 \end{pmatrix} \qquad\qquad (b)\ B = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & 1 \end{pmatrix}$$

7. Consider the following linear system

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 1 \\ 2x_1 + x_2 + 3x_3 &= 2 \\ 3x_2 - x_3 &= 0 \end{aligned}$$

(a) Find the solution of the system by using the Cramer's Rule.

(b) (*Note that the matrix of the coefficients of the system is the matrix A in* $7\,(a)$.) Find the solution of the system using the formula

$$X = A^{-1}b$$

where X is the 3×1 matrix of the unknowns and b is the 3×1 matrix of the right hand side of the system (*Use* A^{-1} *computed in* $7\,(a)$).

8. Find the eigenvalues and the corresponding eigenspaces of the following matrices

(a) $\mathbf{A} = \begin{pmatrix} 5 & 3 \\ -6 & -4 \end{pmatrix}$ (b) $\mathbf{A} = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$ (c) $\mathbf{A} = \begin{pmatrix} 2 & 3 \\ -3 & -4 \end{pmatrix}$

(d) $\mathbf{A} = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ (e) $\mathbf{A} = \begin{pmatrix} 1 & -2 & -1 \\ 0 & 3 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ (f) $\mathbf{A} = \begin{pmatrix} 2 & 1 & 3 \\ 0 & 3 & 1 \\ 0 & 0 & 4 \end{pmatrix}$