Math 351 Homework 6, Fall 2008

Due Wednesday, November 12

1. Given the matrices $A_{4\times 4},\ B_{3\times 4},\ C_{4\times 3},\ D_{3\times 3},\ F_{2\times 3},\ G_{2\times 4}$.
   Determine whether the following operations make sense. If yes what is the size of the resulting matrix?

$$A\cdot B, \qquad B\cdot A, \qquad A\cdot B^{\top} - D, \qquad F\cdot B + 4G$$

$$(A\cdot C\cdot D)^{\top} + 2B, \qquad A\cdot(B^{\top}+C)\cdot D, \qquad (A\cdot G^{\top}) - (C\cdot D\cdot F^{\top}).$$

2. Given

$$A = \begin{pmatrix} 3 & 0 \\ -1 & 5 \end{pmatrix} \quad B = \begin{pmatrix} 4 & -2 & 1 \\ 0 & 2 & 3 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 3 \\ 0 & 1 \\ 2 & 0 \end{pmatrix} \quad D = \begin{pmatrix} 4 & 2 \end{pmatrix} \quad E = \begin{pmatrix} -1 \\ 2 \end{pmatrix}.$$

   Compute (if it is possible)

$$B - C, \qquad B - C^{\top}, \qquad BA, \qquad B^{\top}C^{\top} - (CB)^{\top}, \qquad DAE, \qquad (I_2 - A)^2.$$

3. Verify $(ABC)^{\top} = (C^{\top}B^{\top}A^{\top})$ where

$$A = \begin{pmatrix} 5 & 3 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} -2 & 1 \\ 6 & 4 \\ 5 & 3 \end{pmatrix} \qquad C = \begin{pmatrix} 4 & -1 \\ 1 & 2 \end{pmatrix}$$

4. Let $A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix}$. Show that $A^2 = 5I_3 + 2\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$.

5. Consider

$$M_1 = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{and} \qquad M_2 = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & -\cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

   Compute $P = M_1M_2$. Show that $P^2 = I_3$.

6. (a) If $A = \begin{pmatrix} 4 & 2 \\ -8 & -4 \end{pmatrix}$, check that $A^2 = O$ where O is the zero 2×2 matrix (the square of a non zero matrix can be the zero moatrix!).

   (b) Let $A = \begin{pmatrix} 2 & 1 \\ 6 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ -1 & -1 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$. Check that $AB = AC$ although $B \neq C$.

7. Compute AB and $B^\top A$ where

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 0 & 3 & -1 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 0 & 1 & 0 \\ 0 & 1 & 3 & 2 \\ 1 & -1 & 1 & 0 \end{pmatrix}$$

8. Determine whether each matrix is in row echelon form (REF). If not use raw reduction to put it in REF.

$$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 1 & 2 \\ 1 & 0 & 1 & 2 \end{pmatrix} \quad \begin{pmatrix} 0 & 2 & 1 & 0 & 0 \\ 1 & 3 & -1 & 3 & 2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & -1 & 0 & 0 \\ 0 & 3 & -1 & 3 & 2 \\ 0 & 2 & 0 & 3 & 2 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 2 & 1 & -1 & 0 \end{pmatrix}$$

9. Consider the linear system

$$\begin{aligned} x + 2y + 3z &= 1 \\ 2x + 3y + 4z &= 2 \\ 4x + 7y + 9z &= 1 \end{aligned}$$

   (a) Solve the system

   (b) Replace the third equation by $4x + 7y + 10z = 6$ and show that the new system does not have solutions

   (c) Replace the third equation by $4x + 7y + 10z = 4$ and find all solutions solutions

10. Consider the following linear system

$$\begin{aligned} x_1 + x_2 &= 1 \\ x_2 + x_3 &= 2 \\ x_1 + x_2 + (a^2 - 4)x_3 &= a - 1 \end{aligned}$$

(a) Determine all values of a for which the system has
(i) No solution (ii) A unique solution (iii) Infinitely many solutions

(b) Set $a = 1$. Find the solution of the corresponding system by Gaussian elimination method.

(c) Set a equal to a value that you found in point (a) iii. Find all solutions of the corresponding system.