Math 351-013 Homework Set # 4

Due Monday, October 13

1. Solve the initial value problem

$$y'' - y' - 2y = 0, \quad y(0) = \alpha,\ y'(0) = 2.$$

Then find α so that the solution approaches zero as $t \to +\infty$.

2. Use the method of the reduction of order to find a second solution and then the general solution of the given linear homogeneous equations (note that one solution is provided).

$$x^2y'' - x(x+2)y' + (x+2)y = 0, \qquad y_1(x) = x, \qquad x > 0$$

3. Find the general solution of the homogeneous Euler equation. Find the solution of the initial value problem if an initial condition is given.

   (a) $x^2y'' + 2xy' - 2y = 0$

   (b) $x^2y'' + xy' + 4y = 0 \qquad y(1) = 1,\ \ y'(1) = 0$

   (c) $x^2y'' - xy' + y = 0$

4. Find a particular solution of the nonhomogeneous equation using the method of undetermined coefficients. Write the general solution. Find the solution of the initial value problem if an initial condition is given.

   (a) $y'' - 16y = x^2$

   (b) $y'' + 3y' = xe^x, \qquad y(0) = 1,\ \ y'(0) = -1$

   (c) $y'' - y' - 2y = e^{-x}$

   (d) $2y'' - y' - 3y = \cos(x), \qquad y(0) = 1,\ \ y'(0) = 0$

   (e) $y'' + y = 2\sin(x)$

   (f) $y'' + 3y' = (2 + x) + 3e^{2x}$

5. Find the general solution to the differential equation (use the method of undetermined coefficients to find a solution of the nonhomogeneous equation)

$$y'' - \omega^2 y = e^t - e^{-t}.$$

(**Hint** *Consider first* $\omega \neq 1$ *and then* $\omega = 1$).

6. Find a particular solution of the nonhomogeneous equation using the method of variation of parameters. Write down the general solution.

   (a) $y'' + 9y = 9sec^2(t)$

   (b) $y'' - 2y' + y = \dfrac{e^x}{x^2}$

   (c) $y'' + y = \tan(\mathrm{t})$

7. Consider the initial value problem for the nonhomogeneous Euler equation

$$x^2y'' - 4xy' + 6y = \frac{1}{x}, \qquad x(1) = 0, \;\; x'(1) = 2$$

   (a) Find the fundamental set of solutions for the corresponding homogeneous Euler equation (set the right hand side 0).

   (b) User the method of variation of parameters to find a particular solution of the given nonhomogeneous Euler equation.

   (c) Write the general solution of the nonhomogeneous Euler equation and then solve the given initial value problem.