Math 351- HOMEWORK 2

Due Monday, September 22

1. Find the general solution of the following linear equations

   (a) $\dfrac{dy}{dx} + x^2 y = 0$

   (b) $\dfrac{dy}{dx} - \dfrac{y}{x} = xe^x$

   (c) $t^2 \dfrac{dy}{dt} + 3ty = \dfrac{1}{t}$

   (d) $\dfrac{dy}{dx} = \dfrac{y^2}{1 - 2xy}$ (*Hint: consider x as the unknown function of y*)

2. Solve the following initial value problems. How does the solution behave as $t \to +\infty$?

   (a) $y' + 2ty = 2t, \qquad y(0) = 1$

   (b) $ty' + 2y = t^2 - t + 1, \qquad y(1) = \frac{1}{2}$

   (c) $xy' + y = e^x, \qquad y(1) = 1$

3. Solve the following Bernoulli's equations

   (a) $xy' - 2y = x^3 y^2$

   (b) $\sqrt{y}(3y' + y) = x$

4. Consider an insulated box (a building, perhaps) with internal temperature $u(t)$. According to the Newton's cooling law, u satisfies the differential equation

$$\frac{du}{dt} = -k\left(u - T(t)\right), \qquad k \text{ is a positive constant}$$

   where $T(t)$ is the ambient temperature which we assume varies sinusoidally, i.e. $T(t) = 32 + 5\cos t$. The initial temperature of the box is $u(0) = 75$ (degree). Find $u(t)$. Describe the behavior of $u(t)$ as t becomes large.