Math 351 - Homework 1

Due Monday, September 15

1. Solve the following equations

   (a) $xy' = (1 - y^2)^{1/2}$

   (b) $y' = 2y^2 + ty^2$

   (c) $y' = e^{x+2y}$

2. Solve the following initial value problems

   (a) $y' = 2(1 + x)(1 + y^2) \qquad y(0) = 0$

   (b) $y' + y^3 = 0 \qquad y(0) = y_0$

   (c) $y' = (y - 1)e^x \qquad y(0) = 2$

3. Consider the equation

$$\frac{dw}{dt} = -kt^\alpha w^3, \qquad w(1) = 1 \qquad (1)$$

   where $k > 0$ and α are constant. Find the solution of (1). Be sure to examine the special case $\alpha = -1$.

4. Consider the initial value problem for the autonomous equation

$$\frac{dy}{dt} = 2y - y^2 \qquad y(0) = y_0 \qquad (3).$$

   Solve (3). Determine the critical points and classify each one as asymptotically stable or unstable.

5. Consider the following non-linear equation

$$(*) \quad \frac{dy}{dt} = \frac{t^2 + y^2}{2ty}$$

   (a) Show that, by making the substitution $y = u \cdot t$, (or $u = \frac{y}{t}$) where u is a new unknown function of t, the equation $(*)$ can be transformed into the separable equation

$$(**) \quad t\frac{du}{dt} = \frac{1 - u^2}{2u}.$$

(b) Find the general solution of the equation $(**)$. Then write a formula for the general solution of the equation $(*)$ by substituting back u by $\frac{y}{t}$. Write the solution in explicit form (i.e., solve it for y).

(c) Find the solution of $(*)$ that satisfies the initial condition $y(1) = 2$.

6. A ball with mass $0.1\,kg$ is thrown upward with intial velocity 20m/sec from the roof of a building $20\,m$ high. The air's dragging coefficient k is $0.1\,kg/sec$ and the gravitational acceleration is $g = 10\,m/sec^2$.

   (a) Based on the free fall linear equation $m\frac{dv}{dt} = mg - kv$ write an initial value problem for the velocity $v(t)$ (note that the positive direction is downwards). Solve the initial value problem to find $v(t)$.

   (b) Let $x(t)$ be the distance of the ball from the top of the roof at t. Using the equation $\frac{dx}{dt} = v(t)$ find $x(t)$ knowing that $x(0) = 0$.

   (c) Find the maximum high above the ground that the ball reaches. Find the time that the ball hits the ground.

7. The equation of the motion of a body of mass m falling under the action of the gravitational force mg and aerodynamic drag force $f(v)$, is

$$m\frac{dv}{dt} = mg - f(v).$$

   For high velocity it is found that $f(v) = kv^2$, hence the equation is non linear. Suppose that $m = 1\,kg$, $g = 10\,m/sec^2$ and $k = 2\,kg/sec$ find $v(t)$ if the body is released from rest, i.e $v(0) = 0$. What is the terminal velocity of the body, i.e. the limit of $v(t)$ as $t \to \infty$.