

Math 349 Homework 6 (the last), Fall 2008

Due Tuesday, December 2

1. Problems 27, 28, 29, 38 in the textbook on page 307 (section 4.4).
2. Find $\text{Nul}(A)$, $\text{Col}(A)$, $\text{Row}(A)$, $\text{Nul}(A^\top)$ and verify both relations of Theorem 5.10 in the textbook, where A is given by

$$A = \begin{pmatrix} 1 & -1 & 2 & 2 & 1 \\ 2 & 0 & 2 & 1 & 1 \\ -1 & -1 & 0 & 1 & 0 \end{pmatrix}$$

3. Find the orthogonal projection $\text{proj}_W \mathbf{v}$ of \mathbf{v} onto the subspace W and the orthogonal component $\text{perp}_W \mathbf{v}$ of \mathbf{v} to W :

$$(a) \quad \mathbf{v} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}, \quad \mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad W = \text{span}\{\mathbf{u}_1, \mathbf{u}_2\}.$$

$$(b) \quad \mathbf{v} = \begin{bmatrix} 3 \\ -2 \\ 4 \\ -3 \end{bmatrix}, \quad \mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}, \quad \mathbf{u}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \quad W = \text{span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}.$$

4. Find the orthogonal projection W^\perp .

(a)

$$W = \text{span} \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \right\}.$$

(b)

$$W = \text{span} \{[-1 \ 1 \ -2 \ 1], [3 \ 1 \ -1 \ 0]\}.$$

(c) Use part (a) to find an orthogonal basis for \mathbb{R}^3 that contains the vectors

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}.$$

- (d) Use part (b) to find an orthogonal basis for \mathbb{R}^4 that contains the vectors $[-1 \ 1 \ -2 \ 1]$ and $[3 \ 1 \ -1 \ 0]$.

5. Given the basis for \mathbb{R}^3

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}$$

- (a) Apply the Gram-Schmidt Process to obtain an orthogonal basis for the given basis.
- (b) Normalize the basis found in (a) to obtain an orthonormal basis.
- (c) Express the vector $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ as a linear combination of the basis vectors found in part (b).

6. (a) Find the eigenvalues and the eigenvectors of the following symmetric matrices

$$A = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 3 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

- (b) Orthonormalize the set of the eigenvectors of both matrices.
- (c) Write a diagonalization of the matrices A and B with orthogonal diagonalizing matrix Q

From the textbook do also but **do not turn in**:

- Section 5.1: Problems 2, 5, 6, 9, 14
- Section 5.2: Problems 4, 5, 18, 20