Math 349 Homework 4, Fall 2008

Due Thursday, November 6

1. Let S be the collection of vectors $[x, y, z] \in \mathbb{R}^3$ that satisfy the given property. Prove or disprove that S forms a subspace of $\mathbb{R}^3$

   (a) $x - 2y = z$

   (b) $x \geq 0,\ y \geq 0,\ z = 0$

2. Consider S the collection of vectors in $\mathbb{R}^3$ consisting of vectors of the form

$$\begin{bmatrix} a-b \\ b \\ a+b \end{bmatrix}.$$

   (a) Show that S form a subspace of $\mathbb{R}^3$

   (b) Find a basis for S.

3. Let

$$A = \begin{pmatrix} 1 & 1 & -3 \\ 0 & 2 & 1 \\ 1 & -1 & -4 \end{pmatrix}$$

   (a) Is $\mathbf{b} \in \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ in the column space of A?

   (b) Is $\mathbf{w} = [2\ 4\ -5]$ in the raw space of A?

   (c) Is $\mathbf{v} \in \begin{bmatrix} 7 \\ -1 \\ 2 \end{bmatrix}$ in the null space of A?

4. Find a basis and the dimension for the span of the given vectors

   (a) $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ -1 \\ 6 \end{bmatrix}$

   (b) $[0\ 1\ -2\ 1],\ [3\ 1\ -1\ 0],\ [2\ 1\ 5\ 1]$

5. Find a basis for and the dimension of

$$\mathrm{Nul}\,(A),\ \mathrm{Col}\,(A),\ \mathrm{Row}\,(A),\ \mathrm{Nul}\ \,(A^\top),\ \mathrm{Col}\,(A^\top),\ \mathrm{Row}\,(A^\top).$$

if A is the following matrix:

(a) $A = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 0 & 2 \\ -1 & 3 & -4 \end{pmatrix}$

(b) $A = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 1 & 1 \\ 2 & 3 & 1 & 1 & -1 \end{pmatrix}$

6. (a) Do $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$ form a basis for $\mathbb{R}^3$?

(b) Do $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}$ form a basis for $\mathbb{R}^3$?

(c) Do $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 7 \\ 1 \\ 6 \\ -1 \end{bmatrix}$ form a basis for $\mathbb{R}^4$?

7. Find all possible values of rank(A) a α varies, where $A = \begin{pmatrix} 1 & 2 & \alpha \\ -2 & 4\alpha & 2 \\ \alpha & -2 & 1 \end{pmatrix}$

8. Find the rank and the nullity of

$$A = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 1 & 1 \\ 2 & -1 & 1 & 1 & -1 \\ 1 & 0 & 1 & 0 & -2 \end{pmatrix}$$

9. **Honor's Credits** Do Problems 59, 62, 63, 64 from Section 3.5 of the textbook on page 209.