# MIXED BOUNDARY VALUE PROBLEMS IN INVERSE ELECTROMAGNETIC SCATTERING *

F. CAKONI AND D. COLTON

*Department of Mathematical Sciences*
*University of Delaware,*
*Newark, Delaware 19711, USA*
*E-mail: cakoni@math.udel.edu, colton@math.udel.edu*

We consider the scattering of time harmonic incident plane waves by partially coated perfect conductors and screens (including the case of no coating, i.e. the scatterer is a perfect conductor). Of particular interest to us is the inverse problem of determining the shape of surface impedance from a knowledge of the far field pattern of the electric field. Our analysis is based on a study of mixed boundary value problems for Maxwell's equations .

## 1. Introduction

In this paper we will survey recent results we have obtained on the inverse electromagnetic scattering problem for partially coated perfect conductors and screens. In particular, we are interested in obtaining the shape of the scatterer and material properties of the coating (if it exists) from a knowledge of the time harmonic incident field at a fixed frequency and the electric far field pattern for incident and observation directions on the unit sphere. The difficulty in solving such problems lies in the fact that the material properties of the coating, if it exists, is a priori unknown nor is the connectivity or dimensionality of the scatterer. Such problems are ideally suited to the linear sampling method in inverse scattering theory [7] and in this paper we will outline how the inverse scattering problems of the above type can be solved using this method. For further details we refer the reader to our papers [2], [3], [4] and [5].

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## 2. The Inverse Scattering Problem for Partially Coated Obstacles

Let $D \subset \mathbb{R}^3$ be a bounded region with boundary $\Gamma$ such that $D_e := \mathbb{R}^3 \setminus \overline{D}$ is connected. Each simply connected piece of $D$ is assumed to be a Lipschitz curvilinear polyhedron. We assume that the boundary $\Gamma = \Gamma_D \cup \Pi \cup \Gamma_I$ is split into two disjoint parts $\Gamma_D$ and $\Gamma_I$ having $\Pi$ as their possible common boundary and denote by $\nu$ the unit outward normal defined almost everywhere on $\Gamma$.

The direct scattering problem for the scattering of a time harmonic electromagnetic plane wave by a partially coated obstacle $D$ is to find an electric field $E$ and a magnetic field $H$ such that

$$\begin{aligned} \operatorname{curl} E - ikH = 0 \\ \operatorname{curl} H + ikE = 0 \end{aligned} \tag{1}$$

in $\mathbb{R}^3 \setminus \overline{D}$ and on the boundary $\Gamma$ satisfy

$$\begin{aligned} \nu \times E = 0 \qquad &\text{on} \qquad \Gamma_D \\ \nu \times \operatorname{curl} E - i\lambda(x)(\nu \times E) \times \nu = 0 \qquad &\text{on} \qquad \Gamma_I \end{aligned} \tag{2}$$

where $\lambda(x) \geq \lambda_0 > 0$ is the surface impedance, $\lambda \in L_\infty(\Gamma_I)$,

$$E = E^i + E^s, \qquad H = H^i + H^s \tag{3}$$

and

$$\lim_{r \to \infty} (\operatorname{curl} E^s \times x - ikrE^s) = 0 \tag{4}$$

uniformly in $\hat{x} = x/|x|$ where $r = |x|$. The incident field $E^i, H^i$ is, in general, an entire solution of Maxwell's equations in $\mathbb{R}^3$. In particular we consider electromagnetic plane waves given by

$$\begin{aligned} E^i(x) &:= \tfrac{i}{k} \operatorname{curl} \operatorname{curl} p\, e^{ikx\cdot d} \\ H^i(x) &:= \operatorname{curl} p\, e^{ikx\cdot d} \end{aligned} \tag{5}$$

where $k > 0$ is the wave number, $d$ is a unit vector giving the direction of propagation and $p$ is the polarization vector. Letting

$$L_t^2(\Gamma) : = \left\{ u \in \left(L^2(\Gamma)\right)^3 : \ \nu \cdot u = 0 \text{ on } \Gamma \right\}$$
$$H(\operatorname{curl}, D) := \{ u \in \left(L^2(D)\right)^3 \ : \ \operatorname{curl} u \in \left(L^2(D)\right)^3 \}$$

it is shown in [4] that there exists a unique solution to (1)-(5) such that $E$ is in $H_{loc}(\text{curl}, \mathbb{R}^3 \setminus \overline{D}) \cap \{u: \nu \times u|_{\Gamma_I} \in L^2_t(\Gamma_I)\}$.
In addition to the scattering problem (1)-(5), we will also need to consider the *interior* mixed boundary value problem of finding a solution to

$$\begin{aligned} \text{curl}\, E - ikH = 0 \\ \text{curl}\, H + ikE = 0 \end{aligned} \tag{6}$$

in $D$ such that

$$\begin{aligned} \nu \times E = f \qquad &\text{on} \qquad \Gamma_D \\ \nu \times \text{curl}\, E - i\lambda(x)(\nu \times E) \times \nu = h \qquad &\text{on} \qquad \Gamma_I \end{aligned} \tag{7}$$

for a $f = \nu \times u|_{\Gamma_D}$ where $u \in H(\text{curl}, D)$ such that $\nu \times u|_{\Gamma_I} \in L^2_t(\Gamma_I)$ and $h \in L^2_t(\Gamma_I)$. If $\Gamma_I \neq \emptyset$ it was shown in [4] that there exists a unique solution to (6)-(7) such that $E$ is in $H(\text{curl}, D) \cap \{u: \nu \times u|_{\Gamma_I} \in L^2_t(\Gamma_I)\}$.
We now turn our attention to the inverse scattering problem associated with the direct scattering problem (1)-(5). To this end, we note that for $k > 0$ fixed the radiating solution $E^s$ of (1)-(5) has the asymptotic behavior

$$E^s(x) = \frac{e^{ik|x|}}{|x|} \left\{ E_\infty(\hat{x}, d, p) + O\left(\frac{1}{|x|}\right) \right\} \tag{8}$$

as $|x| \to \infty$, where $E_\infty(\hat{x}, d, p)$ is the electric far field pattern and the dependence on $k$ has been suppressed. It can easily be shown [8] that $\hat{x} \cdot E_\infty(\hat{x}) = 0$. the *inverse scattering problem* we are concerned with is to determine $D$ and $\lambda$ from $E_\infty(\hat{x}, d, p)$ for $\hat{x}, d \in \Omega = \{x: |x| = 1\}$, $p \in \mathbb{R}^3$ and $k$ fixed. We note that it suffices to only consider $\hat{x}, -d \in \Omega_0 \subset \Omega$, i.e. limited aperture far field data and, since $E^s$ depends linearly on $p$, we only need to consider three linearly independent polarizations. However, for the sake of simplicity we will restrict our attention to $\hat{x}, d \in \Omega$ and $p \in \mathbb{R}^3$.

**Theorem 2.1.** *$D$ and $\lambda$ are uniquely determined by $E_\infty(\hat{x}, d, p)$ for $\hat{x}, d \in \Omega$ and $p \in \mathbb{R}^3$.*

**Proof.** The proof follows from a slight generation of the results of [9]. □

Having established the uniqueness of our inverse scattering problem, the next step is to reconstruct $D$ (without of course knowing $\lambda$ a priori). To this end we will use the linear sampling method. We first define the far field operator $F : L^2_t(\Omega) \to L^2_t(\Omega)$ by

$$(Fg)(\hat{x}) := \int_\Omega E_\infty(\hat{x}, d, g(d))ds(d), \qquad \hat{x} \in \Omega \tag{9}$$

and note that $F$ is a compact linear operator. Let $E_{e,\infty}$ be the electric far field pattern of the electric dipole

$$\begin{aligned} E_e(x, z, q) &:= \tfrac{i}{k}\mathrm{curl}_x\, \mathrm{curl}_x\, q\, \Phi(x, z) \\ H_e(x, z, q) &:= \mathrm{curl}_x\, q\, \Phi(x, z) \end{aligned} \tag{10}$$

where

$$\Phi(x, z) := \frac{1}{4\pi} \frac{e^{ik|x-z|}}{|x - z|} \tag{11}$$

and $q \in \mathbb{R}^3$, $z \in \mathbb{R}^3$, are the polarization and the source location, respectively, of the electric dipole. The *far field equation* is

$$Fg(\hat{x}) = E_{e,\infty}(\hat{x}, z, q). \tag{12}$$

In general, no solution exists to the far field equation. However, we can assert the existence of an approximate solution which has, as a function of $z$, growth properties which allow us to determine $D$. In the theorem which follows $E_g$ denotes the electric field of an *electromagnetic Herglotz pair* defined by [8]

$$\begin{aligned} E_g(x) &= \int_\Omega e^{ikx\cdot d} g(d)\, ds(d) \\ H_g(x) &= \tfrac{1}{ik}\, \mathrm{curl} E_g(x) \end{aligned} \tag{13}$$

where $g \in L^2_t(\Omega)$.

**Theorem 2.2.** *Assume that $\Gamma_I \neq \emptyset$ and $\lambda > 0$. Then if $F$ is the electric far field operator (9) corresponding to the direct scattering problem (1)-(5), we have that*

*(1) If $z \in D$ then for every $\epsilon > 0$ there exists a solution $g_\epsilon(\cdot, z) = g_\epsilon(\cdot, z, q) \in L^2_t(\Omega)$ satisfying the inequality*

$$\|Fg_\epsilon(\cdot, z) - E_{e,\infty}(\cdot, z, q)\|_{L^2_t(\Omega)} < \epsilon.$$

*Moreover this solution satisfies*

$$\lim_{z\to\Gamma} \|E_{g_\epsilon(\cdot,z)}\|_X = \infty, \quad \text{and} \quad \lim_{z\to\Gamma} \|g_\epsilon(\cdot, z)\|_{L^2_t(\Omega)} = \infty.$$

*(2) If $z \in D_e$ then for every $\epsilon > 0$ and $\delta > 0$ there exists a solution $g_{\delta,\epsilon}(\cdot, z) = g_{\delta,\epsilon}(\cdot, z, q) \in L^2_t(\Omega)$ of the inequality*

$$\|Fg_{\delta,\epsilon}(\cdot, z) - E_{e,\infty}(\cdot, z, q)\|_{L^2_t(\Omega)} < \epsilon + \delta,$$

*such that*

$$\lim_{\delta\to 0} \|E_{g_{\delta,\epsilon}(\cdot,z)}\|_X = \infty, \quad \textit{and} \quad \lim_{\delta\to 0} \|g_{\delta,\epsilon}(\cdot, z)\|_{L^2_t(\Omega)} = \infty.$$

*Here*

$$\|u\|^2_X = \|u\|^2_{H(curl,\, D)} + \|\nu \times u\|^2_{L^2_t(\Gamma_I)}.$$

The proof of the theorem is based on two key results stated in the following lemmas which are proved in [4]. First we note that by superposition it is easy to see that

$$Fg = -ik\mathcal{B}(E_g)$$

where the operator $\mathcal{B} : H_{loc}(\mathrm{curl}, \mathbb{R}^3)) \cap \{u :\ \nu \times u|_{\Gamma_I} \in L^2_t(\Gamma_I)\} \to L^2_t(\Omega)$ maps the electric incident field $E^i$ to the electric far field pattern of the corresponding scattered solution to (1)-(5) and $E_g$ is the electric field of the electromagnetic Herglotz pair with kernel $g$ given by (13).

**Lemma 2.3.** *The electric field of the interior mixed boundary value problem (6)-(7) can be approximated arbitrarily close by the electric field of an electromagnetic Herglotz pair in $H(curl,\, D) \cap \{u :\ \nu \times u|_{\Gamma_I} \in L^2_t(\Gamma_I)\}$.*

**Lemma 2.4.** *The operator $\mathcal{B}$ is continuous, compact and has dense range. Moreover $E_{e,\infty}(\cdot,\, z, q)$ is in the range of $\mathcal{B}$ if and only if $z \in D$.*

The function $g$ of Theorem 2.2 is sought for by applying Tikhonov regularization and the Morozov discrepancy principle to the far field equation. The relationship between such a regularized solution and the function $g$ has recently been examined by Arens [1] and a numerical study of this approach for determining $D$ has recently been given by Collino, Fares and Haddar [6].

Having determined $D$, the function $g$ of the above theorem can also be used to determine the essential supremum of the surface impedance $\lambda = \lambda(x)$ from a knowledge of the electric far field pattern [2]. In particular, let $E_z$ denote the solution of the interior mixed boundary value problem (6)-(7) with boundary data

$$\begin{aligned} f &= -\nu \times E_e(\cdot,\, z, q) \\ h &= -\nu \times \mathrm{curl}\, E_e(\cdot,\, z, q) + i\lambda(x)(\nu \times E_e(\cdot,\, z, q)) \times \nu \end{aligned} \tag{14}$$

for $z \in B_r \subset D$ where $B_r$ is a ball of radius $r$. In [4] as part of the proof of Theorem 2.2 it is shown that $E_z$ can be approximated in the norm $\|\cdot\|_X$ of the above theorem by the electric field of an electromagnetic Herglotz pair with kernel $ikg$ where $g$ is the above approximate (regularized) solution of the far field equation. Noting that if we define

$$W_z := E_z + E_e(\cdot, z, q) \tag{15}$$

then $\nu \times W_z = 0$ on $\Gamma_D$, the following theorem provides a variational method for determine $\|\lambda\|_{L_\infty(\Gamma_I)}$. Note that $\Gamma_I$ is *not* determined by this method.

**Theorem 2.5.** *Let $\lambda \in L_\infty(\Gamma_I)$ be the surface impedance of the scattering problem (1)-(5). Then*

$$\|\lambda\|_{L_\infty(\Gamma_I)} = \tag{16}$$

$$\sup_{\substack{z_i \in B_r, q \in \mathbb{R}^3 \\ \alpha_i \in \mathbb{C}}} \frac{\sum_{i,j} \alpha_i \overline{\alpha_j} \left[ -\|q\|^2 A(z_i, z_j, k, q) + k \left( q \cdot E_{z_i}(z_j) + q \cdot \overline{E}_{z_j}(z_i) \right) \right]}{2 \| \sum_i \alpha_i (W_{z_i})_T \|^2_{L^2_t(\Gamma)}}$$

*where*

$$A(z_i, z_j, k, q) = \frac{k^3}{6\pi} \left[ 2 j_0(k|z_i - z_j|) + j_2(k|z_i - z_j|)(3\cos^2\phi - 1) \right]$$

*with $j_0$ and $j_2$ being spherical Bessel functions of order 0 and 2, respectively, and $\phi$ is the angle between $(z_i - z_j)$ and $q$.*

The proof of this theorem follows from the following lemmas (for the proofs see [2]).

**Lemma 2.6.** *For every two points $z_1$ and $z_2$ in $D$ and polarization $q \in \mathbb{R}^3$ we have that*

$$2 \int_{\Gamma_I} (W_{z_1})_T \cdot \lambda (\overline{W}_{z_2})_T \, ds = -\|q\|^2 A(z_1, z_2, k, q) + k \left( q \cdot E_{z_1}(z_2) + q \cdot \overline{E}_{z_2}(z_1) \right)$$

**Lemma 2.7.** *Let*

$$\mathcal{E} := \left\{ f \in L^2_t(\Gamma_I) : \begin{array}{c} f = (W_z)_T|_{\Gamma_I} \text{ with } W_z \text{ define by (15),} \\ z \in B_r, \text{ and } q \in \mathbb{R}^3 \end{array} \right\}.$$

*Then $\mathcal{E}$ is complete in $L^2_t(\Gamma_I)$.*

In the particular case where $\lambda$ is a positive constant the formula (16) simplifies to

$$\lambda = \frac{-\frac{k^2}{6\pi}\|q\|^2 + k\mathrm{Re}\,(q \cdot E_{z_0})}{\|(W_{z_0})_T\|^2_{L^2_t(\Gamma)}} \tag{17}$$

where $z_0 \in B_r$.

## 3. The inverse scattering problem for screens

Let $S$ be a bounded, simply connected, orientated, smooth open surface in $\mathbb{R}^3$ with piecewise smooth boundary which does not intersect itself. We consider $S$ as part of the smooth boundary $\partial D$ of some bounded connected open set $D \subset \mathbb{R}^3$ and denote by $\nu$ the normal vector to $\Gamma$ that coincides with the outward normal vector to $\partial D$. We denote by $\nu \times E^+$, $\gamma_T E^+$ and $\nu \cdot E^+|_\Gamma$, ($\nu \times E^-$, $\gamma_T E^-$ and $\nu \cdot E^-|_\Gamma$) the restriction to $S$ of the traces $\nu \times E|_{\partial D}$, $\gamma_T E|_{\partial D}$ and $\nu \cdot E|_{\partial D}$ respectively, from the outside (from the inside) of $\partial D$ where $\gamma_T u := \nu \times (u \times \nu)$ is the tangential component of $u$. The scattering of electromagnetic waves by the screen $S$, under the assumption that $S$ is perfectly conducting, leads to the boundary value problem

$$\begin{aligned} \mathrm{curl}\, E - ikH = 0 \\ \mathrm{curl}\, H + ikE = 0 \end{aligned} \tag{18}$$

in $\mathbb{R}^3 \setminus \overline{S}$ such that

$$\gamma_T E^\pm = 0 \qquad \text{on} \quad S \tag{19}$$

where

$$E = E^i + E^s, \qquad H = H^i + H^s, \tag{20}$$

$E^i$, $H^i$ are given by (5) and $E^s$ satisfies the Silver-Müller radiation condition (4). We refer to this problem as the *screen problem.* The following theorem is proved in [3]:

**Theorem 3.1.** *The screen problem has a unique solution $E, H \in H_{loc}(curl, \mathbb{R}^3 \setminus \overline{S})$.*

We note that the solution of the screen problem always has a singularity near the edge! In particular, even if $S$ is smooth and has smooth boundary, the solution $E, H$ to the screen problem is such that $E, H \in H^{\frac{1}{2}-\epsilon}_{loc}(\mathbb{R}^3 \setminus \overline{S})$

for every $\epsilon > 0$ but $E, H \notin H^{\frac{1}{2}}_{loc}(\mathbb{R}^3 \setminus \overline{S})$! Hence, in order to develop a numerical method for solving the screen problem it is necessary to take into account edge singularities.

In order to formulate the inverse scattering problem associated with (18)-(20), we note that the electric field $E$ again has the asymptotic behavior (8). Hence our inverse problem is to determine $S$ from a knowledge of $E_\infty(\hat{x}, d, p)$ for $\hat{x}, d$ on the unit sphere $\Omega$, three linearly independent polarizations $p \in \mathbb{R}^3$ and $k$ fixed. The following theorem can be established in the same way as the corresponding uniqueness theorem for domains with non-empty interior [9].

**Theorem 3.2.** *The screen $S$ is uniquely determined from a knowledge of $E_\infty(\hat{x}, d, p)$ for all $\hat{x}, d \in \Omega$ and three linearly independent polarizations $p \in \mathbb{R}^3$.*

A reasonable question to ask is if only a finite number of incident waves $d$ suffice to determine uniqueness. In the case of scattering of TM-polarized incident waves by an infinite cylinder (i.e. the Dirichlet crack problem for the two dimensional Helmholtz equation) partial results in this direction have been established by Rondi [10]. However, the case of a perfectly conducting screen in $\mathbb{R}^3$ remains an open problem.

Having established uniqueness, the next step is to reconstruct $S$ from the far field data. We will again do this by using the linear sampling method. To this end we define

$$H^{-\frac{1}{2}}_{div}(S) := \left\{ u \in H^{-\frac{1}{2}}(S) : \nu \cdot u = 0 \quad \mathrm{div}_S\, u \in H^{-\frac{1}{2}}(S) \right\}$$

and let $\tilde{H}^{-\frac{1}{2}}_{div}(S)$ denote those functions in $H^{-\frac{1}{2}}_{div}(S)$ that can be extended by zero to a function in $H^{-\frac{1}{2}}_{div}(\partial D)$. Note that for $u \in H_{loc}(\mathrm{curl}, \mathbb{R}^3 \setminus \overline{S})$ the trace $\nu \times u|_S$ is in $\in H^{-\frac{1}{2}}_{div}(S)$. We again define the far field operator by (9), where now $E_\infty$ is the electric far field pattern for the screen problem, and define the far field equation by

$$(Fg)(\hat{x}) = \Phi^L_\infty(\hat{x}) \tag{21}$$

where

$$\Phi^L_\infty(\hat{x}) := \hat{x} \times \left( \int_L \alpha_L(y)\, e^{-ik\hat{x}\cdot y} ds_y \right) \times x \tag{22}$$

for $L \subset S$ and $\alpha_L \in \tilde{H}_{div}^{-\frac{1}{2}}(L)$. The far field operator corresponding to the screen problem can be factored as

$$(Fg)(\hat{x}) = -ik\mathcal{G}(\nu \times E_g) \tag{23}$$

where the operator $\mathcal{G} : H_{div}^{-\frac{1}{2}}(S) \to L_t^2(\Omega)$ maps $\nu \times E^i|_S$ to the electric field of the far field pattern of the scattered field corresponding to an arbitrary incident field $E^i, H^i$ such that $E^i \in H_{loc}(\text{curl}, \mathbb{R}^3 \setminus \overline{S})$. Here $E_g$ is the electric field of the electromagnetic Herglotz pair with kernel $g$ given by (13). The following lemma was proved in [3]:

**Lemma 3.3.** *The operator $\mathcal{G} : H_{div}^{-\frac{1}{2}}(S) \to L_t^2(\Omega)$ is compact, injective and has dense range. Moreover the range of $\mathcal{B}$ consists of functions $\Phi_\infty^L$ defined by (22) for $L \subset S$ and $\alpha_L \in \tilde{H}_{div}^{-\frac{1}{2}}(L)$.*

Combining this lemma with the fact that any function in $H_{div}^{-\frac{1}{2}}(S)$ can be approximated by $\nu \times E_g$ with respect to the $H_{div}^{-\frac{1}{2}}(S)$-norm we can prove the following theorem.

**Theorem 3.4.** *Let $S$ be a perfectly conducting screen having electric far field pattern $E_\infty$. Then*

*(1) if $L \subset S$ then for every $\epsilon > 0$ there exists a solution $g_\epsilon^L \in L^2(\Omega)$ of the inequality*

$$\|Fg_\epsilon^L - \Phi_\infty^L\|_{L^2(\Omega)} \leq \epsilon.$$

*(2) if $L \not\subset S$ then for every $\epsilon > 0$ and $\delta > 0$ there exists a solution $g_{\epsilon,\delta}^L \in L^2(\Omega)$ of the inequality*

$$\|Fg_{\epsilon,\delta}^L - E_\infty^L\|_{L^2(\Omega)} \leq \epsilon + \delta$$

*such that*

$$\lim_{\delta \to 0} \|g_{\epsilon,\delta}^L\|_{L^2(\Omega)} = \infty \quad \text{and} \quad \lim_{\delta \to 0} \|E_{g_{\epsilon,\delta}^L}\|_{H(curl, B_R)} = \infty$$

*where $E_{g_{\epsilon,\delta}^L}$ is the electric field of the electromagnetic Herglotz pair with kernel $g_{\epsilon,\delta}^L$.*

In order for the above theorem to be of practical value, we need to have the right hand side of the far field equation to be independent of $L$! To this end, we let $L$ degenerate to a point with $\alpha_L$ being an appropriate delta sequence in order to replace $\Phi_\infty^L$ by $E_{e\infty}(\hat{x}, z, q)$, i.e. we now have the same far field equation as we did for a partially coated obstacle with non-empty interior (c.f. (12))! In particular this is the far field equation we use to reconstruct $S$ from $E_\infty$. For numerical examples using this approach, see [3].

The above method for reconstructing the shape of a perfect conducting screen $S$ can also be used to reconstruct a partially coated screen, i.e. one in which one side is coated by a dielectric and the other side is a perfect conductor [5]. In this case the total electric field $E = E^s + E^i$ satisfies the following boundary conditions

$$\begin{aligned} \gamma_T E^- = 0 \qquad & \text{on} \qquad S^- \\ \nu \times \operatorname{curl} E^+ - i\lambda(x)\gamma_T E^+ = 0 \qquad & \text{on} \qquad S^+ \end{aligned} \tag{24}$$

where $\lambda(x) \geq \lambda_0$ and $\lambda \in L_\infty(S)$ and $S^+$ and $S^-$ denote the positive and negative sides of the screen respectively. One can prove [5] that there exists a unique solution of this problem such that $E \in H_{loc}(\text{curl}, \mathbb{R}^3 \setminus \overline{S}) \cap \{u : \ \nu \times u^+ \in L^2_t(S)\}$. But in this case the singularity of the solution near the scree edge is much stronger than in the case of a perfect conductor because roughly speaking the change of the boundary conditions contributes to the edge singularity. In general, even if $S$ is smooth with smooth boundary, the electric field cannot be more regular than $H^{\frac{1}{4}-\epsilon}_{loc}(\mathbb{R}^3 \setminus \overline{S})$ for every $\epsilon > 0$. A detailed analysis of the associated inverse problem shows that Theorem 3.4 is also valid in the case of mixed boundary value problem for screens. Examples of reconstructions based on this theorem for partially coated screens are presented in [5]. However in this case we do not know how to use $g$ to determine the essential supremum of the surface impedance $\lambda(x)$ as we did for partially coated obstacles with non-empty interior.

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