The Identification of Partially Coated Dielectric Objects

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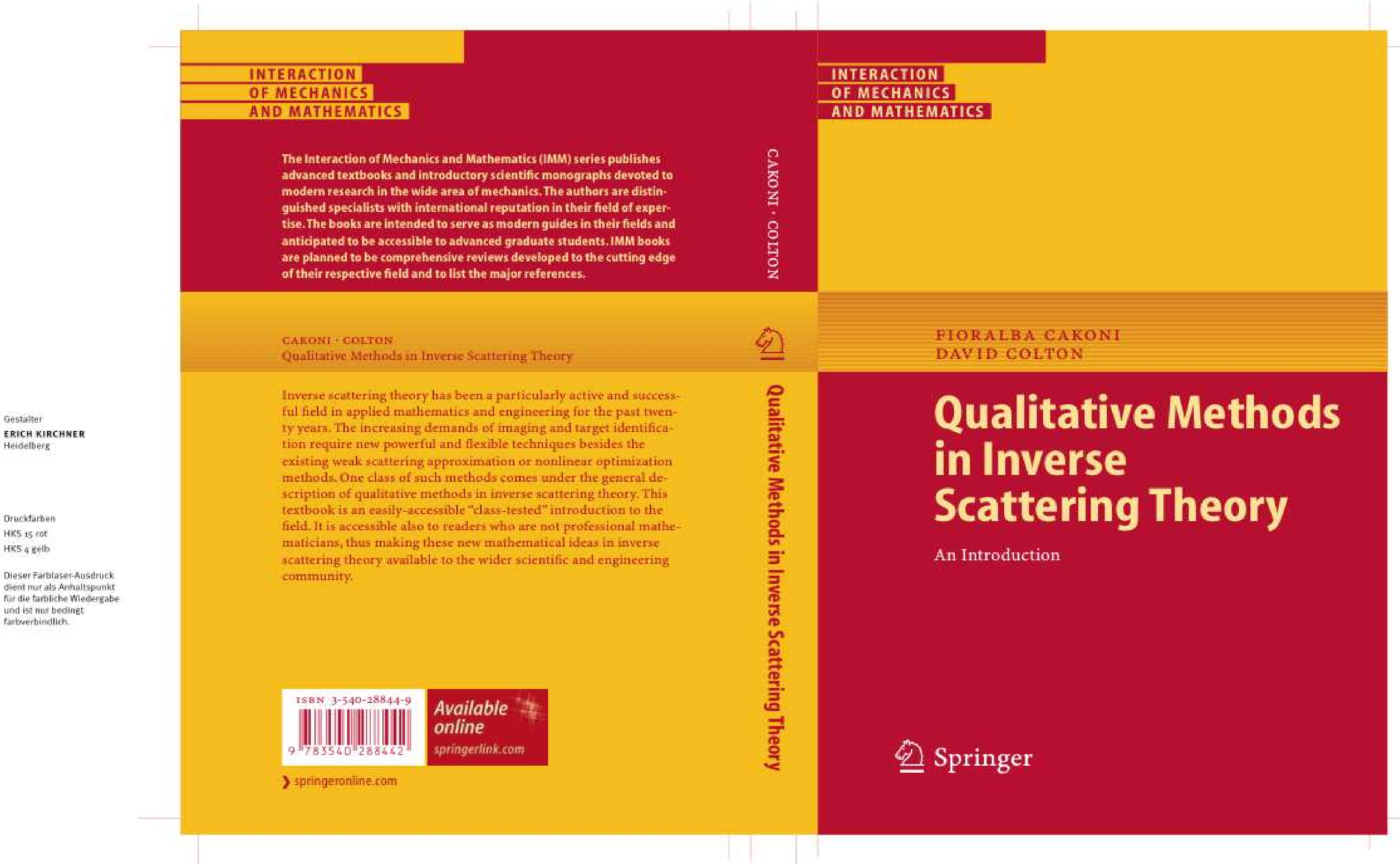
Outlines

- The aim is to **identify (not only detect!) complex scattering objects** using time harmonic electromagnetic radiation.
  - We want to find the shape, and extract information about the physical properties of the object.
  - We want rely on little a priori information and do it in a rather quick and simple way.
  - We have to use multistatic data. Shape reconstruction is not very sharp and not complete recovery of the physical properties is achieved.
- The inversion algorithm will be based on the ”solution” of the equation used by the linear sampling method (*Colton-Kirsch 1996*).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Qualitative Methods in Inverse Scattering Theory
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Qualitative Methods in Inverse Scattering Theory
An Introduction
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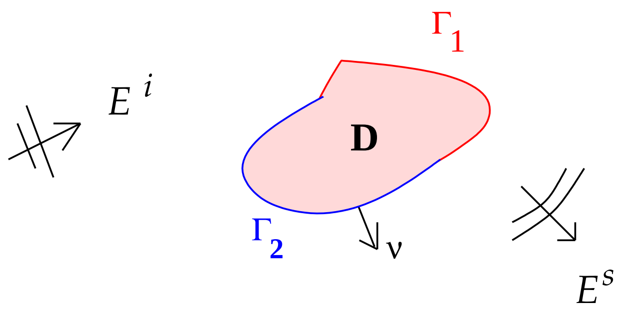
Scattering by an Anisotropic Medium



- The scatterer is an infinitely long cylinder.
- The incident wave is such that the electric field is polarized ⊥ to the cylinder axis.

- The dielectric is orthotropic, i.e. the index of refraction is given by

$$N(x) = \frac{1}{\epsilon_0}\left(\epsilon(x) + i\frac{\sigma(x)}{\omega}\right) = \begin{pmatrix} n_{11} & n_{12} & 0 \\ n_{21} & n_{22} & 0 \\ 0 & 0 & n_{33} \end{pmatrix}.$$

Thus the incident, scattered and interior magnetic fields have the form

$$H^i(0,\ 0,\ u^i), \quad H^{int} = (0,\ 0,\ v), \quad H^s = (0,\ 0,\ u^s)$$

The Forward Problem

$$\nabla \cdot A(x) \nabla v + k^2 v = 0 \quad \text{in} \quad D$$

$$\Delta u + k^2 u = 0 \quad \text{in} \quad \mathbb{R}^2 \setminus \overline{D}$$

$$v - u = 0 \quad \text{on} \quad \partial D_t$$

$$v - u = -i\eta(x) \frac{\partial u}{\partial \nu} \quad \text{on} \quad \partial D_c$$

$$\frac{\partial v}{\partial \nu_A} - \frac{\partial u}{\partial \nu} = 0 \quad \text{on} \quad \partial D$$

$$\lim_{r \to \infty} \sqrt{r} \left(\frac{\partial u^s}{\partial r} - iku^s \right) = 0$$

$$u(x) = u^s(x) + e^{ikx \cdot d}, \quad d \in \Omega := \{x : \ |x| = 1\}, \quad \frac{\partial v}{\partial \nu_A} := \nu \cdot A \nabla v$$

The Forward Problem

We assume:

- ∂D is piecewise smooth.
- In general, A is a symmetric matrix-valued function with piecewise C^1 entries in $\overline{D}$.
- $\mathcal{R}e\left(\overline{\xi}\cdot A\xi\right) \geq \gamma|\xi|^2$ and $\mathcal{R}e\left(\overline{\xi}\cdot A^{-1}\xi\right) \geq \gamma|\xi|^2$, $\gamma > 0$
- $\mathcal{I}m\left(\overline{\xi}\cdot A\xi\right) \leq 0$.
- $\eta \in L_\infty(\partial D_c)$ such that $\eta(x) \geq \eta_0 > 0$

Inverse Scattering Problem

The scattered field u^s has the asymptotic behaviour

$$u^s(x) = \frac{e^{ikr}}{\sqrt{r}} u_\infty(\hat{x}, d) + O\left(r^{-3/2}\right)$$

as $r \to \infty$ where $r = |x|$, $\hat{x} = x/r$, k is fixed and u_∞ is the far field pattern of the scattered field u^s.

The inverse scattering problem is to determine the **shape** D, the **coated part** ∂D_c, the **surface conductivity** η and the **index of refraction** A, from a knowledge of $u_\infty(\hat{x}, d)$ for $\hat{x}, d \in \Omega$ (and possibly for a range of frequencies k).

In fact in the following it suffices to know $u_\infty(\hat{x}, d)$ only for $d \in \Omega_1 \subset \Omega$ and $\hat{x} \in \Omega_2 \subset \Omega$

Uniqueness Theorems

Theorem: (Uniqueness of D)
Assume that either $\bar{\xi}\,\mathcal{R}e\,(A - I)\xi \geq \delta\|\xi\|^2 > 0$ or $\bar{\xi}\,\mathcal{R}e\,(I - A)\xi \geq \delta\|\xi\|^2 > 0$ in D for some δ. Then, D is uniquely determined by $u_\infty(\hat{x}, d)$ for $\hat{x}, d \in \Omega$ and a fixed value of the wave number k.

Proof *by Hähner (2000) for* $\eta = 0$, *Cakoni-Colton (2003) for* $\eta > 0$

The main idea is to use the well-posedness of the corresponding Interior Transmission Problem which will be discussed later in this talk.

Uniqueness Theorems

Theorem: (Uniqueness of η) Given A and D, suppose that for an arbitrary $\Gamma_0 \subset \partial D$ there exists an incident direction d such that $\partial u/\partial\nu \neq 0$ in Γ_0. Then $\eta \in C(\overline{\partial D_c})$ is uniquely determined from $u_\infty(\hat{x}, d)$ for $\hat{x}, d \in \Omega$ and a fixed value of the wave number k.

Proof *by Cakoni-Colton-Monk (2005).*

In the anisotropic case, i.e. $A = a(x)I$, *the reconstruction formula for* η *in Cakoni-Sini-Zeev (to appear) also proves the uniqueness for* $\eta \in C(\overline{\partial D_c})$ *without the assumption that* a *is fixed.*

O'Dell (2006)

Uniqueness Theorems

Uniqueness of A

- If A is a matrix (anisotropic case) it is known that $u_\infty(\hat{x}, d)$ for $\hat{x}, d \in \Omega$ does not uniquely determine A even if it is known for an interval of values of k.

  Gylys-Colwell (1996)

- If $A(x) = a(x)I$ (isotropic case) Gylys-Colwell (1996) has shown that $u_\infty(\hat{x}, d)$ for $\hat{x}, d \in \Omega$ for two different frequencies uniquely determine $a(x)$ provided that $a(x)$ is sufficiently close to a constant.

  Note that in $\mathbb{R}^3$ the assumption that $a(x)$ is sufficiently close to a constant is not needed; see Nachman (1987).

The Far Field Equation

Let $\Phi(x,z) := \frac{i}{4}H_0^{(1)}(k|x-z|)$ which has the far field pattern

$$\Phi_\infty(\hat{x},z) = \frac{e^{i\pi/4}}{\sqrt{8\pi k}}e^{-ik\hat{x}\cdot z}.$$

Define the far field equation

$$(Fg)(\hat{x}) = \Phi_\infty(\hat{x},z), \qquad g \in L^2(\Omega),\ z \in \mathbb{R}^2$$

where $F : L^2(\Omega) \to L^2(\Omega)$ is the far field operator defined by

$$(Fg)(\hat{x}) := \int_\Omega u_\infty(\hat{x},d)g(d)ds(d).$$

The Far Field Equation

For z in D the far field equation has a solution g iff there exist a solution (v_z, w_z) of the interior transmission problem

$$\nabla \cdot A \nabla w_z + k^2 w_z = 0 \quad \textit{and} \quad \Delta v_z + k^2 v_z = 0 \qquad \textit{in} \quad D$$

$$w_z - v_z = \Phi(\cdot, z) \qquad \textit{on} \quad \partial D_t$$

$$w_z - v_z = \Phi(\cdot, z) - i\eta \frac{\partial}{\partial \nu}\left(v_z + \Phi(\cdot, z)\right) \qquad \textit{on} \quad \partial D_c$$

$$\frac{\partial w_z}{\partial \nu_A} - \frac{\partial v_z}{\partial \nu} = \frac{\partial}{\partial \nu}\Phi(\cdot, z) \qquad \textit{on} \quad \partial D$$

such that v_z is a Herglotz wave function $v_g(x) := \int\limits_{\Omega} e^{ikx\cdot d} g(d)\, ds(d)$.

Interior Transmission Problem

Letting $\mathbf{U} = A\nabla w_z - \nabla v_z$ then the solvability of the interior transmission problem follows from the study of the following bilinear form (Cakoni, Colton and Haddar (2007)): For $\mathbf{U}, \psi \in \mathcal{H}_0$

$$\mathcal{A}(\mathbf{U}, \psi) := i \int_{\partial D_c} \eta(\nabla \cdot \mathbf{U})(\nabla \cdot \overline{\psi}) ds +$$

$$\int_D (A^{-1} - I)^{-1} \left(\nabla\nabla \cdot \mathbf{U} + k^2 \mathbf{U}\right) \cdot \left(\nabla\nabla \cdot \overline{\psi} + k^2 A^{-1} \overline{\psi}\right)\, dx$$

$$\mathcal{H}_0 := \left\{ u \in L^2(D),\ \nabla \cdot u \in H^1(D),\ \nu \cdot u|_{\partial D} = 0,\ \nabla \cdot u|_{\partial D_t} = 0 \right\}$$

Definition: Transmission eigenvalues are the values of $k > 0$ for which the homogeneous interior transmission problem (i.e. if $\Phi(\cdot, z) = 0$) has a nontrivial solution.

ITP: What is known!

Assume that in D we have
either C1 $\|\mathcal{R}e\, A^{-1}\|_2 \geq \delta > 1$ or C2 $0 < \beta \leq \|\mathcal{R}e\, A^{-1}\|_2 \leq \delta < 1$.

- The interior transmission problem satisfies the Fredholm alternative in $H^1(D) \times H^1(D)$.
- If $\mathcal{I}m\left(\bar{\xi} \cdot A\xi\right) < 0$ in D then there are no transmission eigenvalues.
- If $\mathcal{I}m\left(\bar{\xi} \cdot A\xi\right) = 0$ then the set of transmission eigenvalues is discrete.
- Any transmission eigenvalue $k > 0$ must satisfies

$$k^2 \geq \frac{\lambda(D)}{\sup_D \|A^{-1}\|_2} \text{ if C1 holds} \quad \text{and} \quad k^2 > \lambda(D) \text{ if C2 holds,}$$

where $\lambda(D)$ is the first Dirichlet eigenvalue of $-\Delta$ on D.

ITP: Open Problems!

- Do we have uniqueness for the solution the interior transmission problem for $\eta \neq 0$?
- Do transmission eigenvalues exists?

The first result in this direction is due to Sylvester and Päivärinta (2007) for the case of $\Delta u + k^2 n(x) u = 0$. They have shown that there exists one transmission eigenvalue providing the contrast $n - 1 > 0$ is large enough.

Remarks

- If k is a transmission eigenvalue and v is a Herglotz wave function then the far field operator $F : L^2(\Omega) \to L^2(\Omega)$ is not injective with dense range, where v, w is the non zero solution of the homogeneous interior transmission problem, i.e. for $\Phi(\cdot, z) = 0$.

- If v_z, w_z is a solution to the interior transmission problem then for every $\epsilon > 0$ there is a Herglotz wave function v_{g_ϵ} such that

$$\|v_{g_\epsilon} - v_z\|_{H^1(D)} < \epsilon.$$

  Colton-Kress (2001)
  Furthermore, for a given $\Omega_0 \subset \Omega$ the above v_{g_ϵ} can be chosen such that g_ϵ is supported on Ω_0, Cakoni-Colton (2003)

Solving the Far Field Equation

We return to the far field equation $(Fg)(\hat{x}) = \Phi_\infty(\hat{x}, z)$. Assume that $k > 0$ is not a transmission eigenvalue.Then

- For $z \in D$ and a given $\epsilon > 0$ there exists a $g_z^\epsilon \in L^2(\Omega)$ such that

$$\|Fg_z^\epsilon - \Phi_\infty(\cdot, z)\|_{L^2(\Omega)} < \epsilon$$

  and the Herglotz wave function $v_{g_z^\epsilon}$ converges in $H^1(D)$ to v_z where v_z, w_z is the solution of the interior transmission problem. Furthermore,

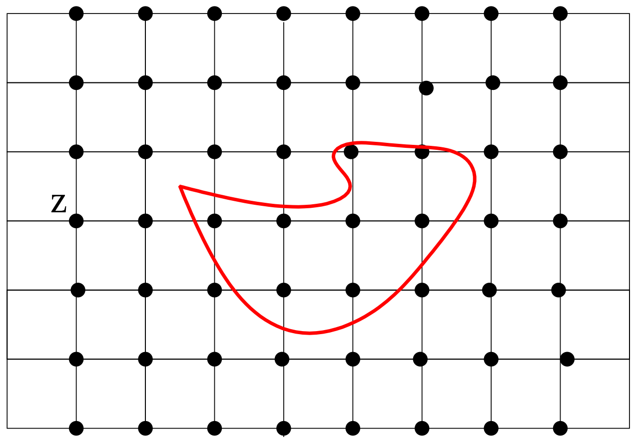
$$\lim_{z \to \partial D} \|v_{g_z^\epsilon}\|_{H^1(D)} = \infty \text{ and } \lim_{z \to \partial D} \|g_z^\epsilon\|_{L^2(\Omega)} = \infty$$

- For $z \in \mathbb{R}^2 \setminus \overline{D}$ and a given $\epsilon > 0$, every $g_z^\epsilon \in L^2(\Omega)$ that satisfies

$$\|Fg_z^\epsilon - \Phi_\infty(\cdot, z)\|_{L^2(\Omega)} < \epsilon \quad \text{is such that} \quad \lim_{\epsilon \to 0} \|v_{g_z^\epsilon}\|_{H^1(D)} = \infty.$$

Indicator Function

Note that for $z \in D$, $v_{g_z^\epsilon}(z) \to v_z(z)$ point-wise.



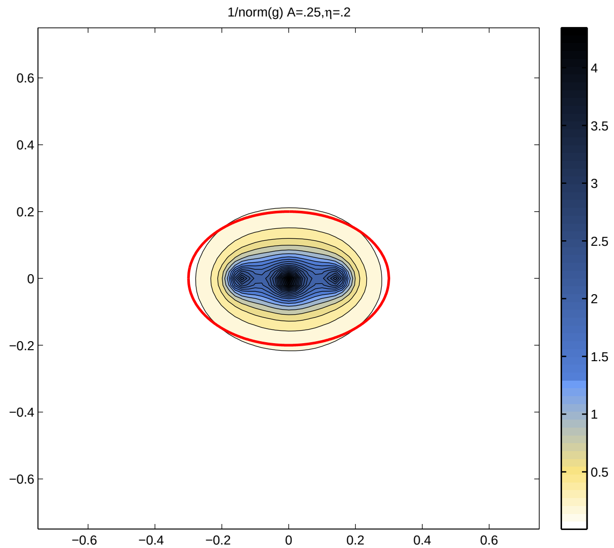
- Construct a grid $\mathcal{G}$.
- For $z_i \in \mathcal{G}$, solve the regularized far field equation $(\alpha I + F^* F) g_{z_i} = \Phi_\infty(\hat{x}, z)$

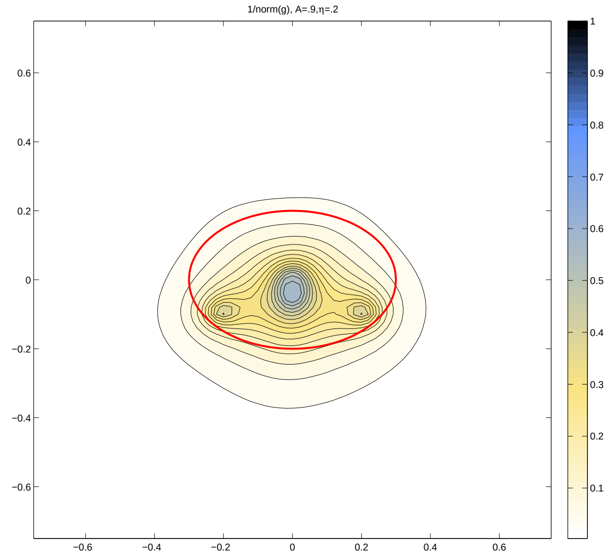
The solution of the inverse problem is based on the use of the regularized solution g_{z_i} of the far field equation and $v_{g_{z_i}}$

Open question: Does g_{z_i} and $v_{g_{z_i}}$ behaves in the same way as the theoretical g_z^ϵ and $v_{g_z^\epsilon}$?
Arens and Lechleiter (2007) have answered positively to this question in the particular case when the far field operator is normal.

Determination of D

We plot level curves $1/\|g_{z_i}\|_{\ell^2} = c$. The boundary is obtained for $c = \theta(\max_z(1/\|g_{z_i}\|)) + (1-\theta)(\min_z(1/\|g_{z_i}\|)),\ 0 < \theta < 1$.





Left it is shown the reconstruction of the partially coated (on the lower half) ellipse with $a = 0.25$ and $\eta = 0.2$. Right is shown the reconstruction of the partially coated (on the lower half) ellipse with $a = 0.9$ and $\eta = 0.2$, Cakoni-Sini-Zeev (2007).

Determination of η

Recall that for $z \in D$, v_{g_z} (where g_z is an approximate solution of the far field equation) converges to v_z where v_z, w_z is the solution of

$$
\begin{aligned}
\nabla \cdot A \nabla w_z + k^2 w_z &= 0 && \text{in} \quad D \\
\Delta v_z + k^2 v_z &= 0 && \text{in} \quad D \\
w_z - v_z &= \Phi(\cdot, z) && \text{on} \quad \Gamma_1 \\
w_z - v_z &= \Phi(\cdot, z) - i\eta \frac{\partial}{\partial \nu}\left(v_z + \Phi(\cdot, z)\right) && \text{on} \quad \Gamma_2 \\
\frac{\partial w_z}{\partial \nu_A} - \frac{\partial v_z}{\partial \nu} &= \frac{\partial}{\partial \nu}\Phi(\cdot, z) && \text{on} \quad \partial D.
\end{aligned}
$$

provided that k is not a transmission eigenvalue.

Determination of η

Assuming that $\mathcal{I}m\left(\bar{\xi}\cdot A\xi\right) = 0$, one can show that (Cakoni-Colton-Monk (2005)) for any $z_1, z_2 \in D$

$$2\int_{\partial D_c} \eta(x)\frac{\partial[v_{z_1}(x)+\Phi(x,z_1)]}{\partial\nu}\,\overline{\frac{\partial[v_{z_2}(x)+\Phi(x,z_2)]}{\partial\nu}}\,ds_x$$

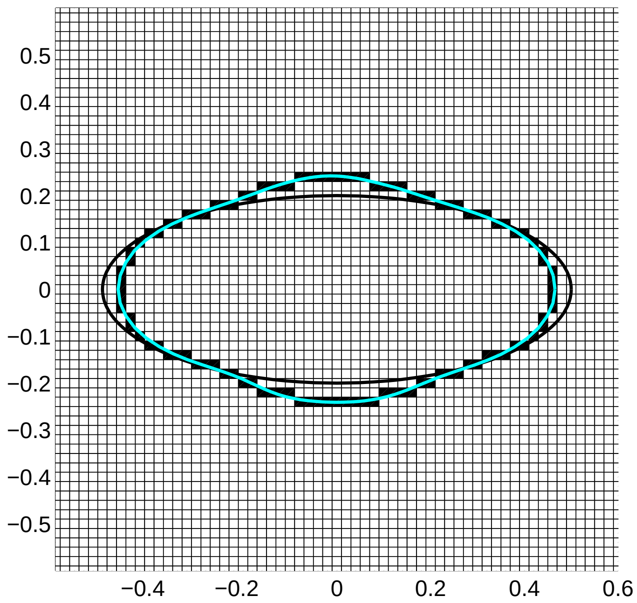
$$= -\frac{1}{2}J_0(k|z_1-z_2|) + iv_{z_1}(z_2) - i\overline{v}_{z_2}(z_1)$$

which allows to estimate $\|\eta\|_{L_\infty(\partial D_c)}$. In particular if η is constant

$$\eta = \frac{-\frac{1}{4} - \mathcal{I}\mathrm{m}\left(v_z(z)\right)}{\|\frac{\partial}{\partial\nu}\left(v_z+\Phi(\cdot\,,z)\right)\|^2_{L^2(\partial D_c)}} \qquad z \in D.$$

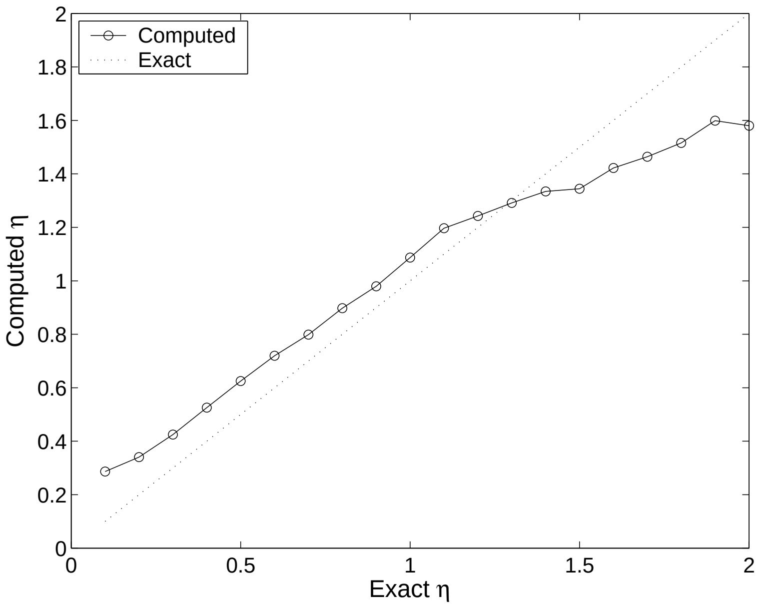
Recall that $v_z(z) \approx v_{g_z}(z)$ where g_z is the regularized solution of the far field equation.

Numerical Examples



Reconstruction of D

(fully coated)



Reconstruction of η

(with reconstructed ∂D)

Complete Identification of the Object

Assume now that $A = aI$, $\mathcal{I}m(a) = 0$, $a \in C^2$ and $\eta \in C^2$.
We consider the solution $g_{z,j}$ of the modified far field equation

$$(Fg)(\hat{x}) = \left[\frac{\partial \Phi(x,z)}{\partial x_j}\right]_\infty \qquad j = 1, 2.$$

$$\nabla \cdot A \nabla w_z + k^2 w_z = 0 \quad \textit{and} \quad \Delta v_z + k^2 v_z = 0 \qquad \textit{in } D$$

$$w_z - v_z = \frac{\partial}{\partial x_j}\Phi(x,z) \qquad \textit{on } \partial D_t$$

$$w_z - v_z = \frac{\partial}{\partial x_j}\Phi(x,z) - i\eta \frac{\partial}{\partial \nu}\left(v_z + \frac{\partial}{\partial x_j}\Phi(x,z)\right) \qquad \textit{on } \partial D_c$$

$$\frac{\partial w_z}{\partial \nu_A} - \frac{\partial v_z}{\partial \nu} = \frac{\partial}{\partial \nu}\left(\frac{\partial}{\partial x_j}\right)\Phi(x,z) \qquad \textit{on } \partial D$$

Complete Identification of the Object

- We do a local analysis of the solution $v_{z,j}, w_{z,j}$ of the corresponding interior transmission problem by comparing them locally at boundary points to the solution of an appropriate problem in the half space which can be solve explicitly.
- A basic ingredient is the point-wise estimate of the Green's function of the interior transmission problem, Nakamura (2007).

Identification of the Coated Part

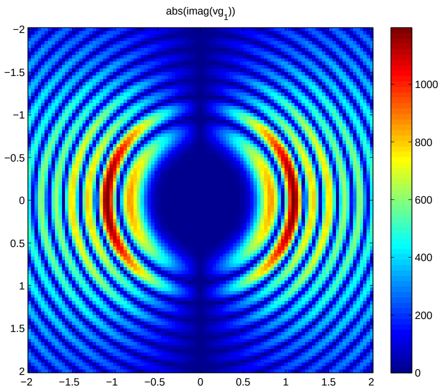
Having reconstructed the boundary ∂D as above we can further obtain, Cakoni-Sini-Zeev (2007)

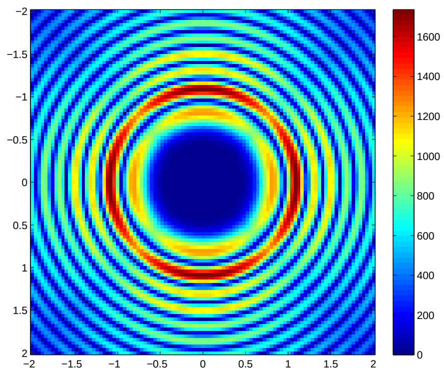
The coated part ∂D_c can be distinguished from the uncoated part ∂D_t by

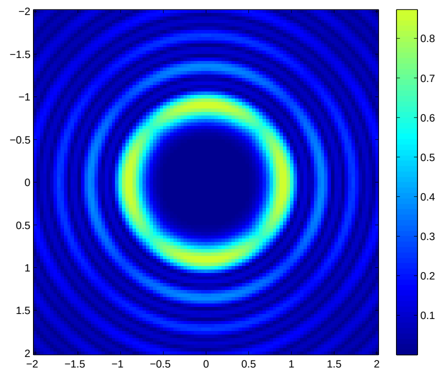
$$\lim_{z \to z_0} \frac{|\mathcal{I}m(v_{z,j}(z))|}{|\ln|(z-z_0)\cdot\nu(z_0)||^s} = \begin{cases} 0 & \text{if } z_0 \in \partial D_t \\ \infty & \text{if } z_0 \in \partial D_c, \end{cases} \qquad s \in (0,\ 1)$$

Recall that $v_{z,j}(z) \approx v_{g_{z,j}}(z)$ where $g_{z,j}$ is the regularized solution of the modified far field equation.

Identification of the Coated Part

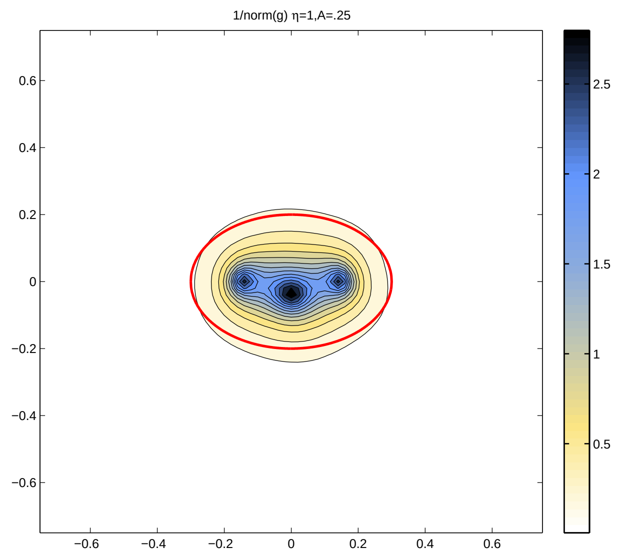


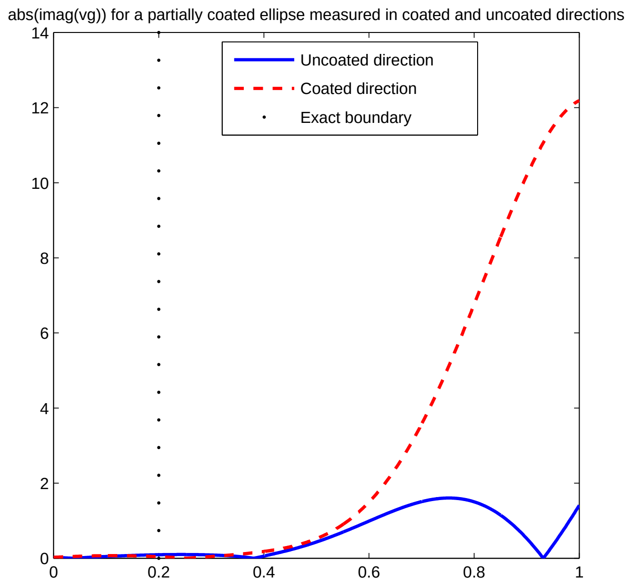




Left plot shows $|\mathcal{I}m(v_{g_{z,1}}(z))|$ for the coated disk with $a = 1.5$ and $\eta = 1$, middle plot shows $|\mathcal{I}m(v_{g_{z,1}}(z))| + |\mathcal{I}m(v_{g_{z,2}}(z))|$ for the coated disk with $a = 1.5$ and $\eta = 1$ and the right plot shows $|\mathcal{I}m(v_{g_{z,1}}(z))| + |\mathcal{I}m(v_{g_{z,2}}(z))|$ for the uncoated disk with $a = 1.5$

Identification of the Coated Part





Left it is shown the reconstruction of the partially coated (on the lower half) ellipse with $a = 0.25$ and $\eta = 1$. Right is shown the behavior of $|\mathcal{I}m(v_{g_{z,1}}(z))|$ at the coated and uncoated boundary points for the reconstructed ellipse on the left.

Determination of η and a

- For $z_0 \in \partial D_c$

$$\eta(z_0) = \lim_{z \to z_0} \frac{-\nu_j(z_0) \ln |(z - z_0) \cdot \nu(z_0)|}{\pi \mathcal{I}m(v_{z,j}(z))} \qquad j = 1, 2.$$

- For $z_0 \in \partial D_t$

$$\frac{a(z_0) - 1}{a(z_0) + 1} = \frac{\nu_j(z_0)}{4\pi \lim_{z \to z_0} \mathcal{R}e(v_{z,j}(z))(z - z_0) \cdot \nu(z_0)} \qquad j = 1, 2.$$

Recall that $v_{z,j}(z) \approx v_{g_{z,j}}(z)$ where $g_{z,j}$ is the regularized solution of the modified far field equation.

Reconstruction of the Normal Vector

$$\nu(z_0) = (\nu_1(z_0), \nu_2(z_0)) \qquad \text{where}$$

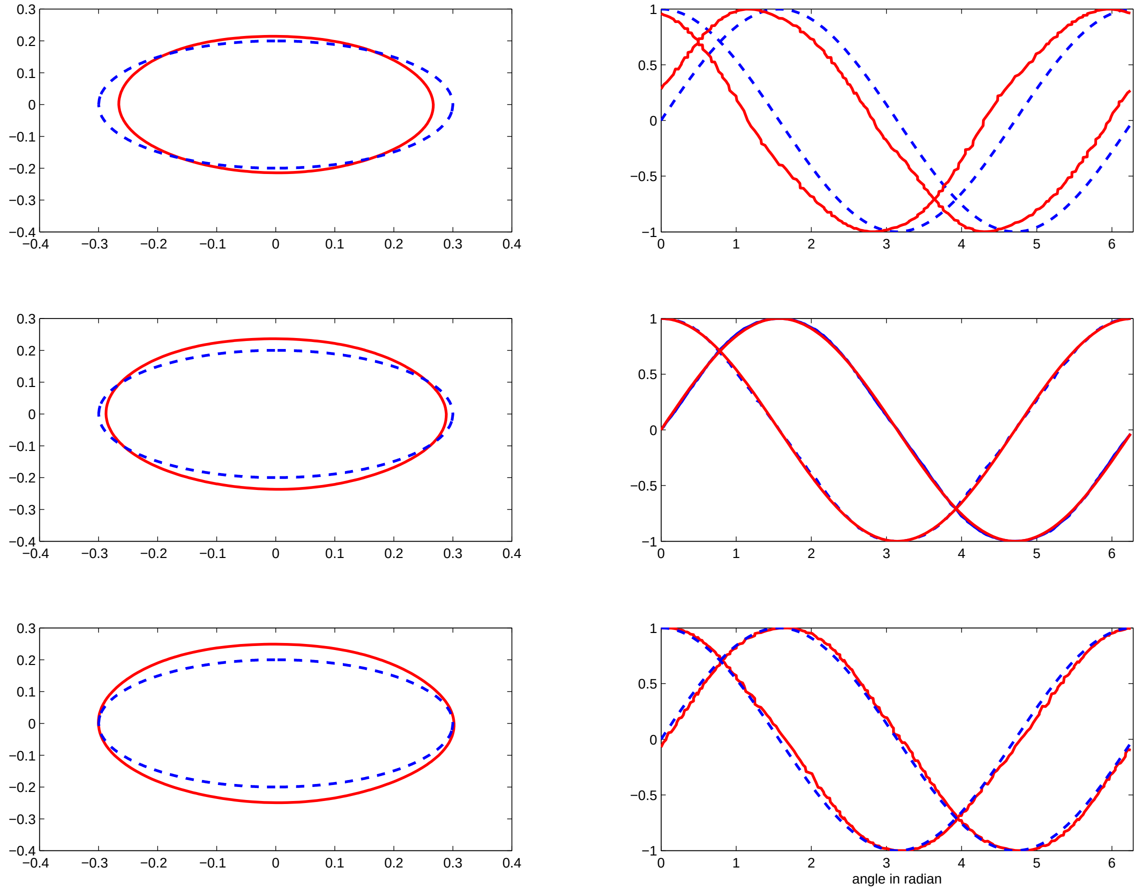
$$\nu_1(z_0) = \lim_{z \to z_0} \pm \frac{\mathcal{R}e(v_{z,1}(z))}{\mathcal{R}e(v_{z,2}(z))} \sqrt{\frac{1}{1 + \left[\frac{\mathcal{R}e(v_{z,1}(z))}{\mathcal{R}e(v_{z,2}(z))}\right]^2}}$$

$$\nu_1(z_0) = \lim_{z \to z_0} \pm \sqrt{\frac{1}{1 + \left[\frac{\mathcal{R}e(v_{z,1}(z))}{\mathcal{R}e(v_{z,2}(z))}\right]^2}}$$

and the sign is chosen so that $\nu(z_0)$ is oriented outside D.

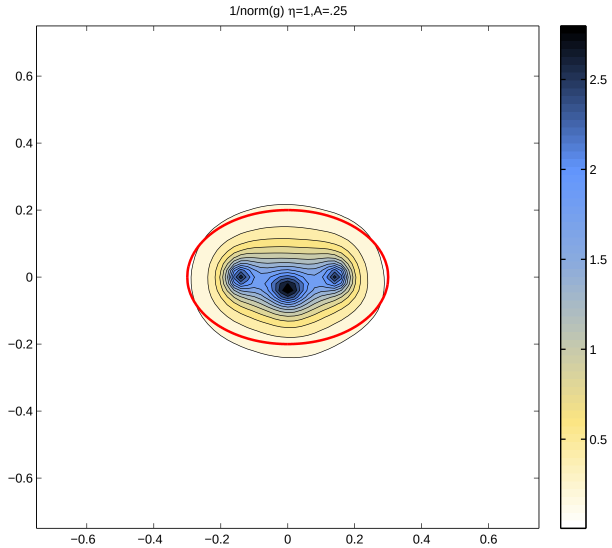
Recall that $v_{z,j}(z) \approx v_{g_{z,j}}(z)$ where $g_{z,j}$ is the regularized solution of the modified far field equation.

Reconstruction of the Normal Vector



Fully coated ellipse with $\eta = 0.2$ and $a = 1.5$.

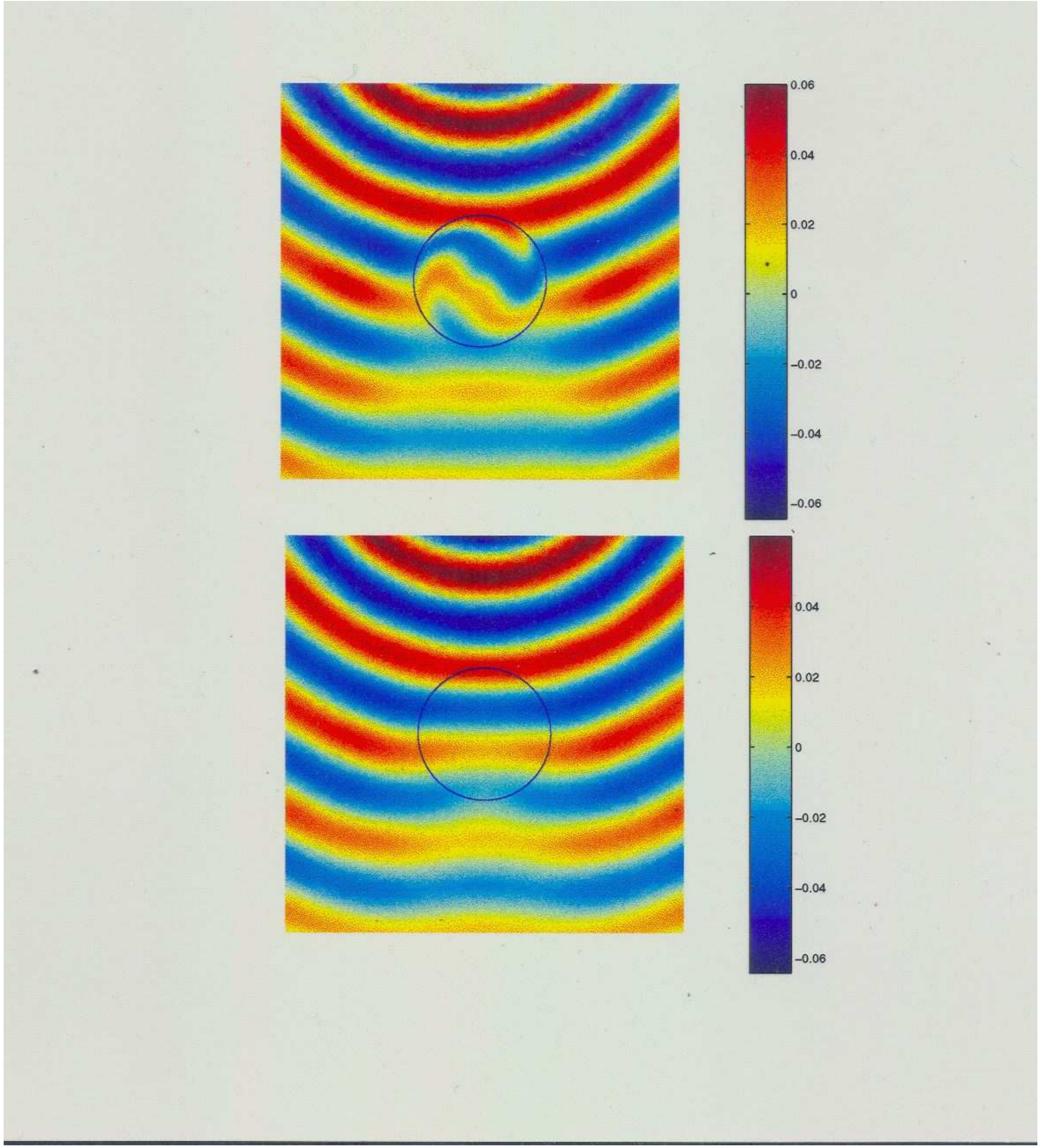
Determination of η and a



Exact η, a	Reconstructed η, a
0.1, 1.1	0.1204, 1.1407
0.1, 1.5	0.1427, 1.6848
0.2, 1.5	0.2892, 1.7904
1, 1.5	1.3147, 1.8601

Left it is shown the reconstruction of the partially coated (on the lower half) ellipse with $a = 1.5$ and $\eta = 1$. The table shows reconstructed values of η and a for the partially coated ellipse

Non-uniqueness for anisotropic media



Interior Transmission Problem

What, if anything, can be said about A from a knowledge of u_∞?

Recall two results we have proven:

- Any transmission eigenvalue $k > 0$ must satisfies

$$k^2 \geq \frac{\lambda(D)}{\sup_D \|A^{-1}\|_2} \text{ if C1 holds} \quad \text{and} \quad k^2 > \lambda(D) \text{ if C2 holds,}$$

  where $\lambda(D)$ is the first eigenvalue of $-\Delta$ on D.

- If k is a transmission eigenvalue and v_z is a Herglotz wave function then the far field operator $F : L^2(\Omega) \to L^2(\Omega)$ is not injective with dense range.

Estimates for A

The first results provides an estimate for the 2-norm of A

- Assume that $\|A^{-1}(x)\|_2 \geq \delta > 1$ for all $x \in D$ and some constant δ. Then,

$$\sup_D \|A^{-1}\|_2 \geq \frac{\lambda(D)}{k^2}$$

- Assume that $0 < \beta \leq \|A^{-1}(x)\|_2 \leq \delta < 1$ for all $x \in D$ and some constants β and δ. Then,

$$k^2 > \lambda(D)$$

where k is the first transmission eigenvalue and $\lambda(D)$ is the first eigenvalue of $-\Delta$ on D.

Computation of Eigenvalues

The second results provide a way to compute the first transmission eigenvalue from the far field.

In particular, the norm of the (regularized) solution to

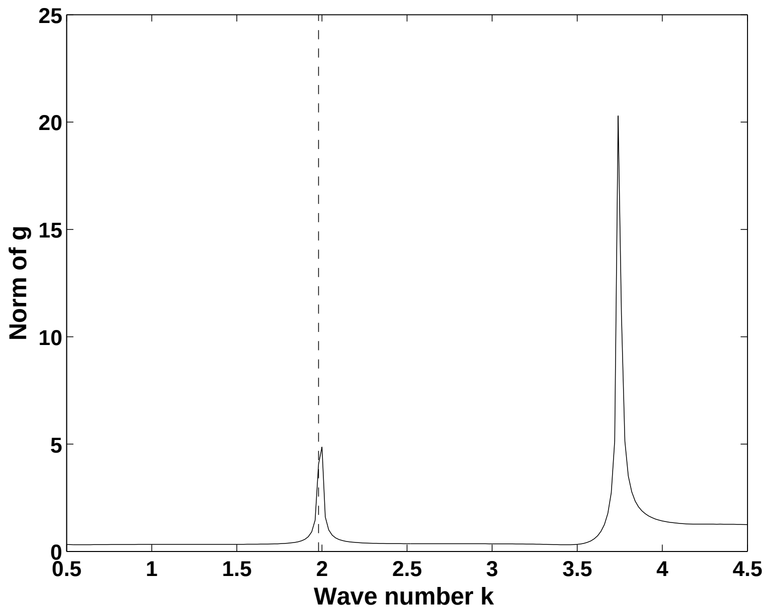
$$(Fg)(\hat{x}) = \Phi_\infty(\hat{x}, z_0) \qquad z_0 \in D$$

should be large for such values of k.

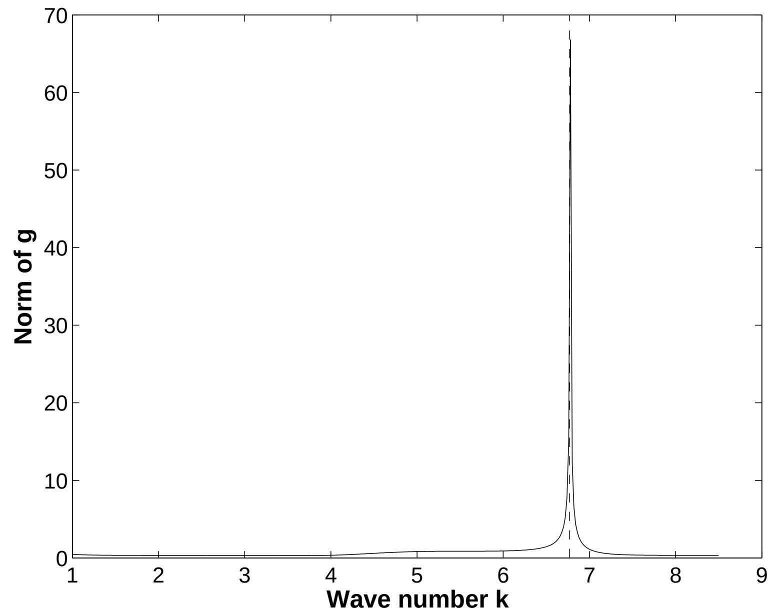
Cakoni-Colton-Haddar (2007)

Numerical Examples

D is a disk of diameter 1, $A^{-1} = nI$



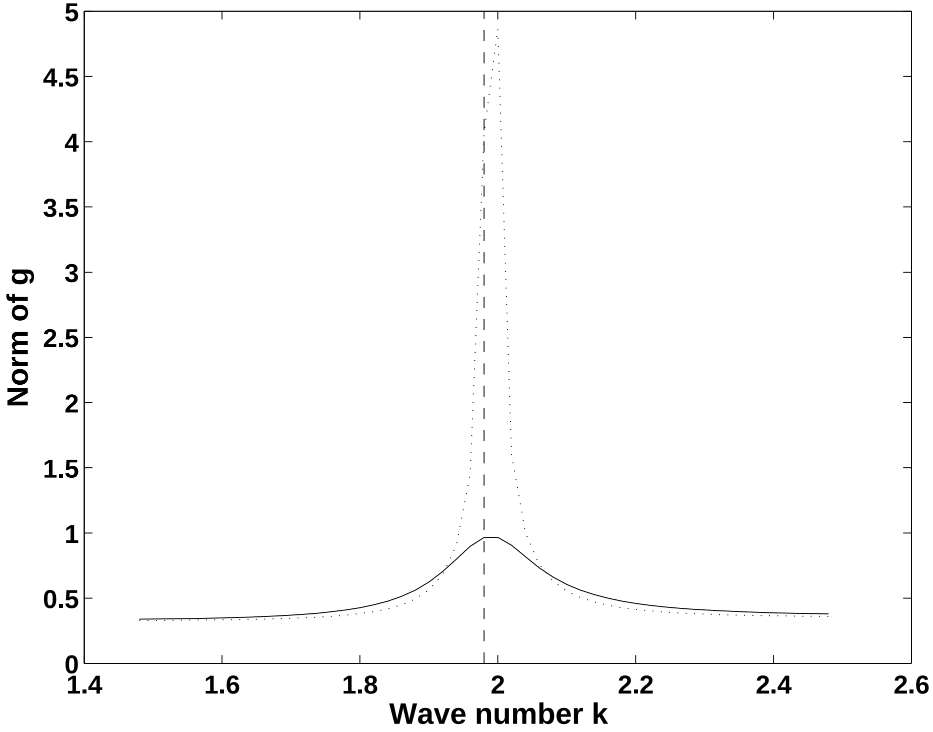
$n = 16$



$n = 4$

Numerical Examples

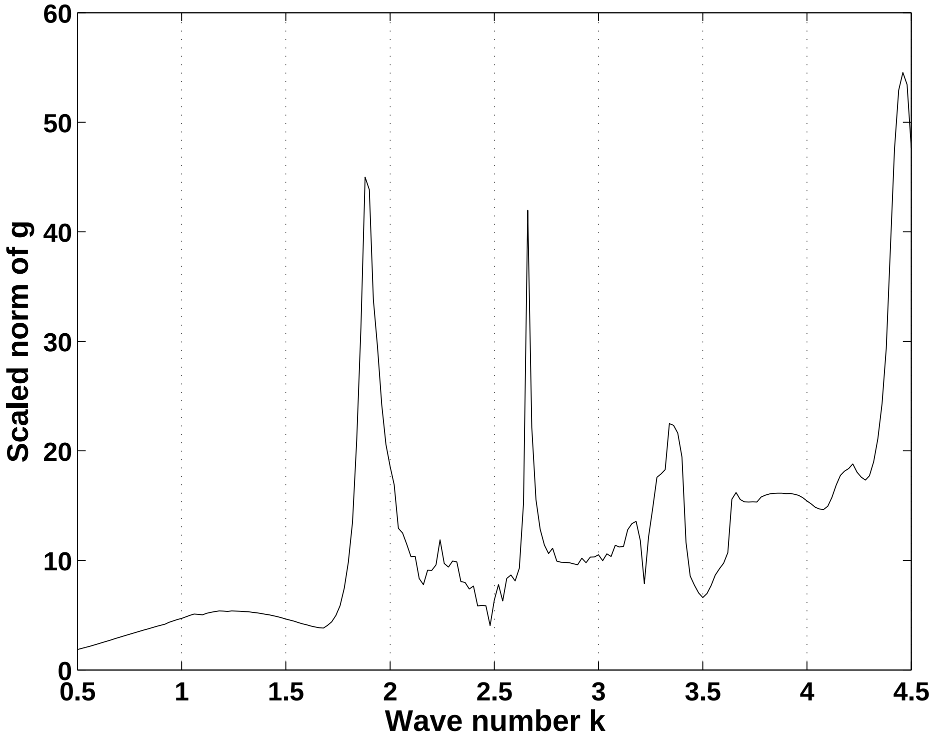
D is a disk of diameter 1.



$$n = 16 + i,\ \eta = 0$$

Numerical Examples

D is the L-shape $= \{[-0.5,\ 0.5] \times [-0.5,\ 0.5]\} \setminus \{]0,\ 0.5] \times]0,\ 0.5]\}$, $A^{-1} = nI$ and $\eta = 0$.



Transmission eigenvalues computed from the far field equation for $n = 16$.

Numerical Examples

D is the L-shape $= \{[-0.5,\ 0.5] \times [-0.5,\ 0.5]\} \setminus \{]0,\ 0.5] \times]0,\ 0.5]\}$,
$A^{-1} = nI$, $\eta = 0$ and $\lambda(D) = 38.6$

n	2.	3.	4.	6.	9.	12.	16.
k_0	15.5	8.1	6.3	4.5	3.3	2.8	2.3
$n_{\min}$	0.2	0.6	1.	1.9	3.5	4.9	7.2

First transmission eigenvalues (k_0)
and lower bounds of the index of refraction $A^{-1} = nI$

Numerical Examples

D is the rectangle $[-0.5,\, 0.5] \times [-0.4,\, 0.4]$