Typically, each of the 24 chapters starts with a short overview. Most results are proved in detail, and for the unproven statements references are given. It's delightful to read this book, and one can find many great results and new proofs for insights into known results. Also, most of the chapters have a set of exercises, which vary in difficulty. One could use this book for a semester course on orthogonal polynomials and not even cover half of it. It also gives a good starting point for a literature search with its 36 pages of references.

Since this subject has a long and interesting history, there are many books on orthogonal polynomials and special functions in one variable, but we compare the book under review only with the recent books by Andrews, Askey, and Roy [1], Gasper and Rahman [2], and Simon [3, 4]. Simon's books are on orthogonal polynomials on the unit circle, and so overlap with Chapter 8. Apart from this overlap, the books are complementary. Orthogonal polynomials also occur in Andrews, Askey, and Roy [1] and Gasper and Rahman [2], but the emphasis is rather different. Andrews, Askey, and Roy emphasize hypergeometric series, and Gasper and Rahman emphasize basic hypergeometric series; in these two books orthogonal polynomials are of secondary importance. Moreover, Ismail's approach to orthogonal polynomials and basic hypergeometric series is much more bottom-up, using ideas arising from orthogonal polynomials, whereas Gasper and Rahman [2] work top-down.

Ismail's book is a great book on orthogonal polynomials containing a lot of information on the general theory and on explicit families of orthogonal polynomials. Starting with elementary theory and ending with research problems, this book is well-suited both for classroom use and for researchers.

REFERENCES

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[3] B. Simon, *Orthogonal Polynomials on the Unit Circle*, Part 1, AMS, Providence, RI, 2005.

[4] B. Simon, *Orthogonal Polynomials on the Unit Circle*, Part 2, AMS, Providence, RI, 2005.

Erik Koelink
Technische Universiteit Delft

Qualitative Methods in Inverse Scattering Theory. *By Fioralba Cakoni and David Colton*. Springer-Verlag, Berlin, 2005. $79.95. viii+227 pp., softcover. ISBN 3-540-28844-9.

The field of inverse problems, and in particular that of inverse scattering, has produced an exciting collection of interesting and practical results over the last few decades which have impacted many academic and industrial communities. Among them, the medical imaging and remote sensing communities have greatly benefited from an assortment of truly insightful mathematical analyses which have produced several powerful algorithms that actually work quite well in the real world. However, as the authors themselves state, there is still the important need to characterize, or identify, anomalous objects in more complex environments, and especially so with limited real-world resources. This need has created a new class of so-called qualitative methods which avoid knowing, or assuming a priori, such things as the boundary conditions satisfied on the interfaces of anomalous inclusions within known structures or, say, on remote objects, which one attempts to probe with either acoustic, elastic, or electromagnetic waves.

In short, the theory of inverse scattering is largely concerned with determining the geometrical and material nature of remotely located or embedded objects by somehow decoding the important features of interest which are themselves encoded in the waves given off, or echoed, by some kind of incident or probing wavefield. The mathematics which largely describes the physics involved in this process is essentially centered around partial differential equations and functional and Fourier analysis, while the implementation of many methods can require much skill and talent in such areas as numerical analysis and the so-called regularization of ill-posed, or unstable, problems. For this reason, it is a daunting task to undertake any text which aims to develop a theoretical platform on which to provide techniques for proving such things as uniqueness theorems for various scattering problems, and at the same time seeks to exhibit and explain viable numerical algorithms. It should be said sooner rather than later that the authors do exceptionally well in this regard!

The book begins with a solid and clear introduction to the foundations of the mathematical tools commonly encountered in the treatment of inverse scattering and, more generally, inverse problems. The first three chapters treat the basic building blocks of functional analysis and Sobolev spaces, ill-posed problems, and the basic scattering process as derived from the time-harmonic Maxwell equations. From both a practitioner's and a new student's point of view, this is a wise and productive way to begin. The initial heat distribution problem on page 27 motivates and illustrates the ill-posed nature of many inverse problems. This self-contained study continues with the development of an ensemble of commonly practiced regularization methods which seek to provide stable solutions to such ill-posed problems. Again, the initial heat problem is revisited and serves as a concrete example of how the truncated singular value decomposition may be employed to treat such unstable "inversions" of observed noisy data.

The brief study of Maxwell's equations and the resulting two-dimensional scattering problem does well in motivating the Helmholtz equation and the basic elements associated with the so-called forward problem of the imperfect conductor. Namely, under certain assumptions about the nature of the boundary of the scatterer and the associated boundary data, when should there exist a unique solution, and how should it depend on the boundary data? The techniques used to prove these results, such as the real-analyticity of the scattered wave fields in each variable, Rellich's lemma, and the so-called unique continuation principle, are instrumental to many other results within the inverse scattering community, and hence it makes much sense to discuss and use them in this foundational chapter.

Chapter 4 continues the consideration of the scattering problem presented in the previous chapter and concentrates on the so-called inverse problem: given the scattering data for the forward problem, what data is sufficient to uniquely determine the scatterer and the impedance condition on its boundary? And, from a practical point of view, how does one attempt to numerically determine, or reconstruct, the scatterer or, say, the impedance function? The answers to both of these questions are presented, and in answering the latter question, the authors give the origins of the so-called linear sampling method, which they continue to apply later in the text to other more complex scattering problems. There is also some discussion on the partial aperture problem, when all the angular data associated with the scattering problem is simply not available.

Chapters 5 and 6 continue along the lines of the previous two chapters. Here, the scatterer is far more complicated in nature and involves a localized orthotropic medium; however, the scattering problem remains two dimensional. Due to the more complex nature of this kind of scattering, the authors build up a well self-contained unit dedicated to the variational treatment of the forward problem, and discuss such cornerstone results as the Lax–Milgram lemma. After establishing uniqueness of the forward problem and delving into an interesting collection of function spaces suited to this problem, again under certain assumptions of the orthotropic medium, they complete the picture by offering results for the accompanying inverse problem of determining such details as the support of the orthotropic medium.

Chapter 7 offers an illuminating commentary and analysis of the so-called factorization method. The authors show its connection with the linear sampling method, how it was motivated, and all the genuinely insightful machinery which went into creating it. It should be said that the authors' methodical description of the factorization method is to be commended. Patient readers are well rewarded for their efforts in following this chapter.

Chapter 8 covers the so-called mixed boundary value problem in a variety of settings, i.e., when some section of the boundary of the scatterer satisfies an impedance boundary condition, or possesses a surface conductivity, while the remainder satisfies a homogeneous Dirichlet condition. The chapter offers numerous interesting uniqueness results as well as several numerical reconstructions of the support of the scatterer in addition to the impedance and surface conductivity parameters. Also, the forward and inverse scattering problems for cracks are considered. These numerical results do well in unifying the more theoretical framework presented earlier in the text.

The final chapter focuses on some of the aspects of the three-dimensional electromagnetic scattering problem and those of the partially coated scatterer, or mixed boundary value problem. Uniqueness of the forward problem is established along with some careful consideration of the function space in which such a solution should exist. The three-dimensional electromagnetic version of the linear sampling method is also briefly discussed, as are some results, or lack thereof, for uniquely reconstructing the orthotropic inhomogeneous medium.

All in all, the authors do exceptionally well in combining such a wide variety of mathematical material and in presenting it in a well-organized and easy-to-follow fashion. This text certainly complements the growing body of work in inverse scattering and should well suit both new researchers to the field as well as those who could benefit from such a nice codified collection of profitable results combined in one bound volume.

STEVEN KUSIAK
Massachusetts Institute of Technology

Numerical Solution of Partial Differential Equations. Second Edition. *By K. W. Morton and D. F. Mayers.* Cambridge University Press, Cambridge, UK, 2005. \$45.00. xiv+278 pp., softcover. ISBN 0-521-60793-0.

The numerical solution of partial differential equations is a subject that I find notoriously difficult to teach. Should one emphasize the often deep connections between the continuous and the discrete models? Or go to the other extreme and teach software engineering issues?

If one considers it important to give students at least an intuitive understanding of basic numerical concepts like truncation errors, stability, and convergence, the book under review is a good text.

First and foremost, the text is very well written. The authors take great care in keeping the presentation at an elementary level. Most but not all of the material is very classical: Basic explicit and implicit difference schemes for the heat equation, for conservations laws, and for Poisson's equation are presented clearly. In addition, there are sections on finite volume methods for simple evolution equations and finite element methods for simple elliptic equations. A chapter on the iterative solution of large linear systems with basic material is also included.

As a theoretical tool of stability analysis, the discrete maximum principle is most often employed, though an introduction to the Fourier analysis of difference schemes, for periodic constant-coefficient problems, is also presented. Fourier analysis is then illustrated by the computation and discussion of amplification factors for model problems.

There is additional material, less well known: A short section on one-dimensional equations originating from higher-dimensional problems written in polar coordinates. The authors discuss how to deal with the singularity at $r = 0$ when employing a difference scheme. Another more substantial section is on Hamiltonian systems and symplectic integration schemes. This is rather recent material in the PDE context. There are quite a few other topics which the authors briefly sketch, such as Harten's TVD concept and the modified equation analysis pioneered by Yanenko and Shokin. The reader gets a taste of the issues involved but then finds *this is well beyond the scope of this book*.

Some criticism: Occasionally the emphasis on keeping the book at an elementary level has led to inaccuracies. For example, in Chapter 6 the Dirichlet problem for Poisson's equation in a square $\Omega = (0,1) \times (0,1)$ is considered, $u_{xx} + u_{yy} + f(x,y) = 0$ in