University of Delaware
Department of Mathematical Sciences
Math 810 Asymptotic and Perturbation Methods 09S
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Homework 5: Due 4/15/09

1. Work thru the details of the expansion of $K(x^*, \epsilon)$ that gives the result just after 2.2.106 in Kevorkian and Cole.

2. Consider the two-point boundary value problem

$$u'' + \frac{2}{r}u' + \epsilon u u' = 0, \quad 1 \leq r < \infty, \quad 0 < \epsilon \ll 1,$$

with boundary conditions

$$u(1) = 0, \text{ and } \lim_{r\to\infty} u(r) = 1.$$

This is a singular perturbation problem, even though the small parameter doesn't multiply the highest derivative. It is a model problem for steady fluid flow past a sphere. Find the two term approximation to the solution; that is, find a uniformly valid solution where the neglected terms are $o(\epsilon)$. Hint: take a look at section 5.2 of Hinch.

3. The dimensional form of the damped driven Duffing equations may be written as

$$M\frac{D^2Y}{DT^2} + B\frac{DY}{DT} + K_1 Y + K_3 Y^3 = F(t).$$

Here Y is the dimensional displacement (say of a mass), M is the mass, B is the damping coefficient, K_0 is the linear spring constant and K_3 is the coefficient of the nonlinear part of the spring behavior, and $F(T)$ is the forcing. For this problem we make the particular choice $F(t) = F_0 \cos(\Omega T)$.

   (a) Nondimensionalize this equation by changing variables using the choices $t = \Omega_N T$ and $y = Y/A$, with A being a characteristic amplitude and $\Omega_N = \sqrt{K_1/M}$ is the natural frequency of the free linear oscillator. In the course of this nondimensionalization, define $\epsilon = K_3 A^2/K_1$, $f = F_0/(K_3 A^3)$ and $\beta = (\sqrt{K_1/M} BA)/(K_3 A^3)$. Explain the physical meaning of these parameters; assume ϵ is small.

   (b) Next, the forcing term will have the form $\epsilon f \cos(t\Omega/\Omega_N)$; choose $\Omega/\Omega_N = 1 + \epsilon\omega$ where $\omega = (1)$ and explain the meaning of ω and this forcing.

   (c) Explain how we could define f and β differently and still have them be $O(\epsilon)$ if the nonlinear term is negligible in the original differential equation.

§4.3 of Kevorkian and Cole may be helpful if you're having problems with the above.