University of Delaware
Department of Mathematical Sciences
Math 810 Asymptotic and Perturbation Methods 09S
R. J. Braun
Homework 4: Due 3/18/09

1. Consider
$$I(x) = \int_x^\infty e^{-t^4}\,dt.$$
(a) Find the three-term expansion of this integral for $x \to 0$ by writing
$$I(x) = \int_0^\infty e^{-t^4}\,dt - \int_0^x e^{-t^4}\,dt,$$
and expanding the latter integral.
(b) Find the 2-term expansion for $I(x)$ with $x \to \infty$.

2. Use integration by parts to show that
$$I(x) = \int_a^b f(t)e^{x\phi(t)}\,dt,\ x \to \infty$$
and
$$I(x;\epsilon) = \int_a^{a+\epsilon} f(t)e^{x\phi(t)}\,dt,\ x \to \infty$$
differ by an exponentially small term. Assume
$$\max_{a \le t \le b} \phi(t) = \phi(a),$$
$\phi'(a) \neq 0$, $f(a) \neq 0$ and that $f(t)$ has a Taylor expansion about $t = a$.

3. Find the leading behavior of
$$I(x) = \int_0^{\pi/2} \sqrt{\sin t}\, e^{-x \sin^4 t}\,dt.$$

4. Verify 2.2.78 through 2.2.85(b) in Kevorkian and Cole.