

Math 428 Numerical and Algorithmic Solution of Differential Equations 08S
Project 4. Due Friday, 5/16/05, 4:30pm.
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Your name: _____

In this project you will examine the numerical solution of the boundary value problem

$$u'' + \lambda e^u = 0, u(0) = u(1) = 0, 0 < x < 1.$$

You will find the solution as a function of the parameter λ in small increments and use the solution from the previous version of λ as an initial guess for the next. This idea is called continuation, and this is a very simple form of it.

You will proceed by using the function `bvp4c` with continuation. An example of continuation is given in the last section of Chapter 10.

For the current project, we will find solutions for $0 < \lambda < \approx 3.5$ and see how the solutions vary with λ . Note that there are two solutions for each λ in this interval, and no solutions for larger λ .

1. Modify the text example to add a loop or two which sends the appropriate values of the parameter λ to the ode function. I suggest finding the solution for increasing λ up to 3.5; You will need a similar loop for decreasing λ over similar values because there are two solutions for each value of λ (provided it is not too large).
2. Plot the $\|y(x)\|_\infty$ vs λ when the maximum value of λ is 3.5. Also plot $y(x)$ for a just few values of λ . How is the solution changing along this curve?
3. Try maximum values of 3, 3.55 and 3.6 for λ as well and plot $\|y(x)\|_\infty$ vs λ in each case. What happens when λ becomes too large? This type of singularity which occurs near $\lambda = 3.55$ is called a fold, and the point at which it occurs is called a limit point.
4. Interpret the meaning of this curve.

Hand in this sheet, a one page summary of your results and answers to any questions above, your plots, and the matlab codes you used.

The project is individual (outside of discussing the general approach you're using or want to use); sharing of code is not permitted.