

Math 428 Numerical and Algorithmic Solution of Differential Equations 08S

R. J. Braun

Department of Mathematical Sciences

University of Delaware

**Honors Project 1. Due 4/17/08, 3:30pm.**

In this project you will reproduce some of the efforts to compute 10 digits for the extreme value of the following integral:

$$I(\alpha) = \int_0^2 [2 + \sin(10\alpha)]x^\alpha \sin(\alpha/(2-x)) dx.$$

You will attempt to treat the integral with (i) the mid point rule to get a rough picture of the problem, (ii) double exponential quadrature together with a change of integration path to get on answer for  $\alpha = \pi/4$  and (iii) use double exponential quadrature together with the secant method to find the value of  $\alpha$  that maximizes  $I(\alpha)$ .

You are expected to use Chapter 9 of the book “The SIAM 100-Digit Challenge” by Bornemann *et al*; I will give you a copy of that chapter.

1. Implement a code to use the midpoint method code to estimate  $I(\alpha)$  on the interval  $\alpha = [0, 1]$ . Plot the part of the integral that is independent of  $x$  on the same plot, as the value of the integral without that factor. You should see that the  $I(\alpha)$  is maximized near  $\pi/4$ .
2. Use the double exponential quadrature method with the mapping of the chapter (p. 189) and the change of integration path (p. 182). You should be able to get many digits of accuracy. You can check your result using `quad` if you wish.
3. By embedding your double exponential quadrature in a function, you can then use the secant method to do the rootfinding to get the largest value of  $I$ . Note: for root finding, you want the zero of something, and finding the root of the derivative with respect to  $\alpha$  is what is needed; see p. 186.
4. Note that you can find the exact integral in Maple if you want; see section 9.6.
5. What are the advantages and disadvantages of each method?

Hand in a brief report with the following outline: (1) introduction, (2) description of methods, (3) results, (4) conclusions. Include your code(s) as an appendix to the report. Be convincing with your data; act like your job depends on it. The project is individual (outside of discussing the general approach you’re using or want to use); sharing of code is not permitted.