Project 1. Due 4/16/08, 3:30pm.

In this project you will examine the numerical integration of the integral

\[ I = \int_{-1}^{1} \frac{100x^2 \cos(\pi x/2)}{\sqrt{1-x^2}} \, dx. \]

You will attempt to treat the integral with (i) double exponential quadrature, (ii) the midpoint method, (iii) the Matlab built-in `quad`, and (iv) Gaussian integration with special weights and nodes. You will examine the error behavior in each case.

1. Use Maple to get the exact answer; use it to compute the error in your approximations.

2. Use the double exponential quadrature function `dequad` to evaluate this function for different \( \delta \) and \( h \). Convince me or any other knowledgeable reader that your code is correct with appropriate computed evidence. What is the largest \( \delta \) and \( h \) for which you can get to about machine precision?

3. Try using the midpoint method by starting with \( h = 0.1 \) and reducing it by an order of magnitude in each attempt. How close can you get? What goes wrong?

4. Try `quad`. What goes wrong?

5. Use the special weights and nodes given in class, namely \( w(1 : n) = \pi/n \) and \( x_j = \cos(\pi * (2 * (1 : n) - 1)/n) \) for integrands of the form

\[ I = \int_{-1}^{1} \frac{f(x)}{\sqrt{1-x^2}} \, dx \]

with the quadrature rule \( I_n = \sum w_j f(x_j) \). Choose \( f(x) \) for the quadrature rule appropriately.

6. What are the advantages and disadvantages of each method? Why might most numerical analysts think that the double exponential method is superior to the special Gauss method, though the results on this particular problem may not suggest that?

Hand in a brief report with the following outline: (1) introduction, (2) description of methods, (3) results, (4) conclusions. Include your code(s) as an appendix to the report. Be convincing with your data; act like your job depends on it. The project is individual (outside of discussing the general approach you’re using or want to use); sharing of code is not permitted.