Math 426/CISC 410 08F, All Sections

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Homework 9 Solutions, Hints and Answers

Problem 4.3.1. The ratios below lead one to the hypothesis that $\beta = |\lambda_1/\lambda_2|$ where λ_1 is the smallest eigenvalue and λ_2 is the next largest. I used the absolute value to be consistent with the text's discussion, but didn't take off any points regarding that.

```
>> Prob4_3_1

ans =
   0.100000000000000
ans =
  Columns 1 through 3
   0.499997426900280   0.500000960319059   0.499998869303925
  Columns 4 through 6
   0.499999272015949   0.499997769577533   0.499995910873903

ans =
   0.190000000000000
ans =
  Columns 1 through 3
   0.954657963528205   0.954460672723885   0.954157789294416
  Columns 4 through 6
   0.953930372977706   0.953686822377336   0.953471954551683

ans =
   0.150000000000000
ans =
  Columns 1 through 3
   0.750033310452803   0.750035041826787   0.750021228506441
  Columns 4 through 6
   0.750018434710601   0.750012562010486   0.750010049412731

ans =
  -0.100000000000000
ans =
  Columns 1 through 3
   0.499999004188412   0.500001272679431   0.499997813475183
  Columns 4 through 6
   0.500004187200427   0.499991718602662   0.500016501112390
>> type Prob4_3_1.m

% Script: Prob4_3_1.m
% explore the rate constant for QR iteration
```

```
V = randn(6);
for jj = [0.1 0.19 0.15 -0.1]

lambda = [1 -0.75 0.6 -0.4 0.2 jj];
D = diag(lambda);
A = V*D/V;
Ak = A;
maxiter = 30;
approx = [];
for k=1:maxiter
    [Q,R] = qr(Ak); Ak = R*Q; approx = [approx Ak(end,end)];
end
format long

disp(' ')
lambda(end)
abs(approx(maxiter/2+10:maxiter)-lambda(end))...
    ./abs(approx(maxiter/2+9:maxiter-1)-lambda(end))

end
>> diary
```

4.4.1. We know that $A\mathbf{x} = \lambda\mathbf{x}$; now subtract $sI\mathbf{x}$ from both sides to get $A\mathbf{x} - sI\mathbf{x} = \lambda I\mathbf{x} - sI\mathbf{x}$. Simplifying gives $(A - sI)\mathbf{x} = (\lambda - s)\mathbf{x}$; this is the form of an eigenvalue problem, with a matrix times a vector giving the same thing as a scalar times a vector. Thus we've shown that the matrix $A - sI$ has eigenvalues $\lambda - s$ and it has the same eigenvectors as A.

4.4.2(e). The smallest eigenvalue is about -0.263 from `eig(F)`; the eigenvalues of the bottom right 2x2 matrix are 1 and 7. We might expect to get 1 or 6.52 for eigenvalues of F first, but running `qrshift` shows that -0.263 is obtained and the convergence appears to be faster than quadratic, and doesn't fit cubic or 2.5 power either. It is clear that the convergence is rapid, however.

```
>> type Prob4_4_2

F = [4 3 2 -1; 3 4 2 -1; 2 2 4 3; -1 -1 3 4 ]
lam = eig(F)
gamma = qrshift(F)
format short e
err = abs(gamma - lam(1))
ratio2 = err(2:end)./err(1:end-1).^2
ratio3 = err(2:end)./err(1:end-1).^3
ratio3 = err(2:end)./err(1:end-1).^2.5

>> Prob4_4_2
```

```
F =
     4     3     2    -1
     3     4     2    -1
     2     2     4     3
    -1    -1     3     4
lam =
 -2.6309e-001
  1.0000e+000
  6.5248e+000
  8.7383e+000
gamma =
 -2.4490e-001 -2.6309e-001 -2.6309e-001 -2.6309e-001
err =
  1.8188e-002  9.1275e-010  4.0523e-015  4.0523e-015
ratio2 =
  2.7591e-006  4.8640e+003  2.4677e+014
ratio3 =
  1.5170e-004  5.3290e+012  6.0897e+028
ratio3 =
  2.0458e-005  1.6100e+008  3.8765e+021
```

4.5.2(ac). Here's a diary file where hess is used.

```
>> Prob4_5_2

A =
    10     3     7
     8     5     2
     6     3     8
P =
  1.0000e+000            0            0
            0 -8.0000e-001 -6.0000e-001
            0 -6.0000e-001  8.0000e-001
H =
  1.0000e+001 -6.6000e+000  3.8000e+000
 -1.0000e+001  8.4800e+000 -1.6400e+000
            0 -2.6400e+000  4.5200e+000
Test how to get H
ans =
  2.2642e-015
C =
     1     2     3     4
     4     3     2     1
     7     5     3     1
     1     3     5     7
P =
```

```
  1.0000e+000            0            0            0
            0 -4.9237e-001 -1.3391e-002 -8.7029e-001
            0 -8.6164e-001 -1.3391e-001  4.8954e-001
            0 -1.2309e-001  9.9090e-001  5.4393e-002
H =
  1.0000e+000 -4.0620e+000  3.5351e+000 -5.4393e-002
 -8.1240e+000  6.9091e+000 -1.5527e+000  3.1870e+000
            0 -5.8497e+000  6.0909e+000  6.1546e-001
            0            0            0 -3.3307e-016
Test how to get H
ans =
  2.2443e-015
>> type Prob4_5_2.m

% Prob4_5_2
% part (a)
A = [10 3 7; 8 5 2; 6 3 8]
[P,H] = hess(A)
disp('Test how to get H')
norm(P'*A*P - H)

%part(c)
C = [ 1 2 3 4; 4 3 2 1; 7 5 3 1; 1 3 5 7]
[P,H] = hess(C)
disp('Test how to get H')
norm(P'*C*P - H)
```