Math 426/CISC 410 08F, All sections

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Homework 4 Solutions, Hints and Answers

Problem 1.6.2. Note that log is denoting the natural log here. Here is one way to solve this problem. See the web page for a published version of this problem.

```
% Script: Prob1_6_2.m
% Math 426/Cisc 410

format compact;  % get rid of white space
% part (a)
disp(' ')
disp(' Problem 1.6.2')
disp(' part (a) ')
disp(' The condition number is f''(x) = 1/sqrt(x^2-1)')
disp(' The conditioning appears to be fine as abs(x) increases. ')

% part (b)
disp(' ')
disp(' part (b) ')
format long;
t = -4:-4:-16;
x = cosh(t);
tt = log(x-sqrt(x.^2 - 1));
errorb = abs(t-tt);
disp(' ')
disp(' g(x) = log(x-sqrt(x.^2-1)) ')
disp(' t          x                g(x)           error  ')
disp('---------------------------------------------------')
for k = 1:4
      disp(sprintf('%3.0f    %7.4e    %16.12g  %8.5e',t(k),x(k),tt(k),errorb(k)))
end

% part c
disp(' ')
disp(' part (c) ')
ttt = -2*log( sqrt((x+1)/2) + sqrt((x-1)/2) );
errorc = abs(t-ttt);
disp(' ')
disp(' h(x) = -2 log( sqrt((x+1)/2) + sqrt((x-1)/2) ) ')
disp(' t          x                h(x)           error  ')
disp('---------------------------------------------------')
for k = 1:4
      disp(sprintf('%3.0f    %7.4e    %16.12g  %8.5e',t(k),x(k),ttt(k),errorc(k)))
```

```
end

% part d
disp(' ')
disp(' part (d) ')
disp(' The formula of part (b) is unstable; the formula of part (c) is stable. ')
disp(' The argument of the log contains a difference of two closely spaced' )
disp('    numbers causing subtractive cancelation! Part (c) avoids this.  ')
```

Problem 2.1.2. The product of Ax is an m-vector that contains numbers ranging from 0 to 3. The number in each row indicates how many of the words are in each document; a larger number indicates more relevance to the search.

Problem 2.1.3. Try it in Matlab.

Problem 2.1.4. (a) Use `C = [1; 0; -2]'` and compute `A=B*C`.
(b) Use `C=[0 0 0 1; 0 0 1 0; 0 1 0 0; 1 0 0 0]` and get the desired result from `A=C*B`.
(c) `A = C'*B*C` with `C=[1; 1; 1]`.

Problem 2.2.1. (a) Consider $AX = B$; we can write $X = [\mathbf{x}_1\ \mathbf{x}_2\ \ldots\ \mathbf{x}_n]$ where the $\mathbf{x}_i$, $i = 1, 2, \ldots, n$, are the columns of X, and $B = [\mathbf{b}_1\ \mathbf{b}_2\ \ldots\ \mathbf{b}_n]$ where the $\mathbf{b}_i$ are the columns of B. The j-th column of AX is $A\mathbf{x}_j$ and this column vector must equal the j-th column of B, $\mathbf{b}_j$. Thus $A\mathbf{x}_j = \mathbf{b}_j$, $j = 1, 2, ..., p$. We can find X by solving these p systems, one for each column of X.
(b) We can find A^{-1} by treating it like X from part (a). We can solve for each column j of A^{-1} by solving the system with right hand side $\mathbf{e}_j$, $j = 1, 2, \ldots, n$. This is computationally better than evaluating a formula for A^{-1}.

Problem 2.2.4. Choosing $b = [0\ \alpha]^T$, $\alpha \neq 0$, will result in the system having no solution. This is easy to see because the column space of A has only vectors with zero in the second place.

Problem 2.2.6(b). We use the system from 2.2.5(b), and Matlab as below.

```
>> L = [4 0 0 0; 1 -2 0 0; -1 4 4 0; 2 -5 5 1]
L =
     4     0     0     0
     1    -2     0     0
    -1     4     4     0
     2    -5     5     1
>> b = [-4 1 -3 5]'
b =
    -4
     1
    -3
     5
>> x = forwardsub(L,b)
x =
```

```
    -1
    -1
     0
     2
>> L*x
ans =
    -4
     1
    -3
     5
>> % L*x is the same as b, so the solution is correct.
```

Problem 2.2.8(b). `>> U = [3 1 0 6; 0 -1 -2 7; 0 0 3 4; 0 0 0 5]`

```
U =
     3     1     0     6
     0    -1    -2     7
     0     0     3     4
     0     0     0     5
>> b = [4 1 1 5]'
b =
     4
     1
     1
     5
>> x = backsub(U,b)
x =
   -3.3333
    8.0000
   -1.0000
    1.0000
>> U*x-b
ans =
     0
     0
     0
     0
>> % The residual is zero, so the answer is correct
```

Problem 2.3.1(a). `U = [ 2 3 4; 0 -1 2; 0 0 -2]` and `L=[1 0 0; 2 1 0; 2 -2 1]`; we can write

$$L = I + 2\mathbf{e}_2\mathbf{e}_1^T + 2\mathbf{e}_3\mathbf{e}_1^T - 2\mathbf{e}_3\mathbf{e}_2^T.$$