Homework 6. (Solutions to selected problems)

Math 353 Section 12, Fall 2008, University of Delaware

1. 4.1.8. (a) Substitute the data points into $y = p_1 + p_2 x$ and we get
   $0 = p_1 + 0p_2$
   $3 = p_1 + 1p_2$
   $3 = p_1 + 2p_2$
   $6 = p_1 + 5p_2$
   or, in matrix form,
$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 5 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 3 \\ 6 \end{pmatrix}.$$
   The normal equations are
$$\begin{pmatrix} 4 & 8 \\ 8 & 30 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} 12 \\ 39 \end{pmatrix}$$
   and solving it we get coefficients $p_1 = 0.8571$ and $p_2 = 1.0714$ for the best line.

2. 4.2.2. (b) Substituting the data into $F_3(x) = p_1 + p_2 \cos(2\pi x) + p_3 \sin(2\pi x)$ and $F_4(x) = q_1 + q_2 \cos(2\pi x) + q_3 \sin(2\pi x) + q_4 \cos(2\pi x)$ and we get

k	$y_k = F_3(x_k)$	$y_k = F_4(x_k)$
1	$4 = p_1 + p_2$	$4 = q_1 + q_2 + q_4$
2	$2 = p_1 + p_2 \cos(\pi/3) + p_3 \sin(\pi/3)$	$2 = q_1 + q_2 \cos(\pi/3) + q_3 \sin(\pi/3) + q_4 \cos(2\pi/3)$
3	$0 = p_1 + p_2 \cos(2\pi/3) + p_3 \sin(2\pi/3)$	$0 = q_1 + q_2 \cos(2\pi/3) + q_3 \sin(2\pi/3) + q_4 \cos(4\pi/3)$
4	$-5 = p_1 - p_2$	$-5 = q_1 - q_2 + q_4$
5	$-1 = p_1 + p_2 \cos(4\pi/3) + p_3 \sin(4\pi/3)$	$-1 = q_1 + q_2 \cos(4\pi/3) + q_3 \sin(4\pi/3) + q_4 \cos(8\pi/3)$
6	$3 = p_1 + p_2 \cos(5\pi/3) + p_3 \sin(5\pi/3)$	$3 = q_1 + q_2 \cos(5\pi/3) + q_3 \sin(5\pi/3) + q_4 \cos(10\pi/3)$

   or, in matrix form,
$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & \cos(\pi/3) & \sin(\pi/3) \\ 1 & \cos(2\pi/3) & \sin(2\pi/3) \\ 1 & -1 & 0 \\ 1 & \cos(4\pi/3) & \sin(4\pi/3) \\ 1 & \cos(5\pi/3) & \sin(5\pi/3) \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 0 \\ -5 \\ -1 \\ 3 \end{pmatrix}$$
   and
$$\begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & \cos(\pi/3) & \sin(\pi/3) & \cos(2\pi/3) \\ 1 & \cos(2\pi/3) & \sin(2\pi/3) & \cos(4\pi/3) \\ 1 & -1 & 0 & 1 \\ 1 & \cos(4\pi/3) & \sin(4\pi/3) & \cos(8\pi/3) \\ 1 & \cos(5\pi/3) & \sin(5\pi/3) & \cos(10\pi/3) \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 0 \\ -5 \\ -1 \\ 3 \end{pmatrix}.$$
   The normal equations are respectively,
$$\begin{pmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 12 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 6 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix} = \begin{pmatrix} 3 \\ 12 \\ 0 \\ -3 \end{pmatrix}$$

and we have the best fit functions

$$F_3(x) = 0.5 + 4\cos(2\pi x), \quad F_4(x) = 0.5 + 4\cos(2\pi x) - \cos(4\pi x).$$

Least square errors are for $F_3(x):\ \sqrt{5.5}$ and for $F_4(x):\ \sqrt{2.5}$.

3. 4.2.4. (b) Substituting the data into the linearized model $\ln y = \ln p_1 + p_2 x$ and we get
$\ln(10) = \ln(p_1) + 0p_2$
$\ln(5) = \ln(p_1) + 1p_2$
$\ln(2) = \ln(p_1) + 2p_2$
$\ln(1) = \ln(p_1) + 3p_2$
or, in matrix form,

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} \ln p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} \ln(10) \\ \ln(5) \\ \ln(2) \\ 0 \end{pmatrix}.$$

The normal equations are

$$\begin{pmatrix} 4 & 6 \\ 6 & 14 \end{pmatrix} \begin{pmatrix} \ln p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} 4.6052 \\ 2.9957 \end{pmatrix}$$

and by solving it we have $p_1 = 10.2257$ and $p_2 = -0.7824$.

4. 4.4.6.

(a)

$$Jr = \begin{pmatrix} e^{c_2 t_1} & c_1 t_1 e^{c_2 t_1} & 1 \\ e^{c_2 t_2} & c_1 t_2 e^{c_2 t_2} & 1 \\ e^{c_2 t_3} & c_1 t_3 e^{c_2 t_3} & 1 \end{pmatrix}$$

(b)

$$Jr = \begin{pmatrix} t_1^{c_2} & c_1 t_1^{c_2} \ln t_1 & 1 \\ t_2^{c_2} & c_1 t_2^{c_2} \ln t_2 & 1 \\ t_3^{c_2} & c_1 t_3^{c_2} \ln t_3 & 1 \end{pmatrix}$$