Home work 3. (Solutions to selected problems)
Math 353 Section 12, Fall 2008, University of Delaware

Exercises.

1. Exercise 1.1.2.
   (a) Let $f(x) = x^5 + x - 1$. Then it has a root in $[0, 1]$.
   (b) Let $f(x) = \sin x - 6x - 5$. Then it has a root in $[-1, 0]$.
   (c) Let $f(x) = \ln x + x^2 - 3$. Then it has a root in $[1, 2]$.

2. Exercise 1.1.5.
   (a) Let $f(x) = x^4 - x^3 - 10$. Then it has a root in $[2, 3]$.
   (b)
$$N = \left[\frac{\ln(10^{-10})}{\ln 2}\right] = 33 \quad \text{iterations.}$$

3. Exercise 1.2.2.
   (a)
$$g'(x) = \frac{2-2x}{x^3} \quad \Rightarrow \quad g'(1) = 0 < 1 \quad \text{FPI is locally convergent.}$$
   (b)
$$g'(x) = -\sin x \quad \Rightarrow \quad g'(\pi) = 0 < 1 \quad \text{FPI is locally convergent.}$$
   (c)
$$g'(x) = 2e^{2x} \quad \Rightarrow \quad g'(0) = 2 > 1 \quad \text{FPI is divergent.}$$

4. Exercise 1.3.4.
   (a)
$$f'(x) = 2x\sin(x^2) + 2x^3\cos(x^2) \;\Rightarrow\; f'(0) = 0,\; f''(x) = 2\sin(x^2) + 10x^2\cos(x^2) - 4x^4\sin(x^2) \;\Rightarrow\; f''(0) = 0,$$
$$f'''(x) = 24x\cos(x^2) - 36x^3\sin(x^2) - 8x^5\cos(x^2) \;\Rightarrow\; f'''(0) = 0 \text{ but } f^{(IV)}(0) \neq 0$$
   the multiplicity of $x = 0$ is 4.
   (b) The forward error is $|0 - 0.01| = 10^{-2}$ and the backward error is $|f(0.01)| = 10^{-8}$.

5. Exercise 1.4.2.
   (a) $f(x) = x^3 + x^2 - 1 \;\Rightarrow\; f'(x) = 3x^2 + 2x$. Therefore,
$$x_{n+1} = x_n - \frac{x_n^3 + x_n^2 - 1}{3x_n^2 + 2x_n} = \frac{2x_n^3 + x_n^2 + 1}{3x_n^2 + 2x_n}$$
   and we find $x_1 = 0.8, \quad x_2 = 0.7568181818182$
   (b) $x_1 = 0.2$ and $x_2 = 0.2786885246$.

(c) $f(x) = 5x - 10 \quad \Rightarrow \quad f'(x) = 5$, thus $x_{n+1} = x_n - f(x_n)/5$ and we find $x_1 = 2$ which is the solution.

6. Exercise 1.4.5.
First note, $f(x) = 8x^4 - 12x^3 + 6x^2 - x = x(2x-1)^3$ therefore, $x = 0$ is a single root and $x = 1/2$ is a multiple root with multiplicity 3. For the single root $x = 0$: Newton's Method converge quadratically and Bisection Method converge linearly, so we choose Newton's Method. For the multiple root $x = 1/2$, both methods converge linearly, but rate of Bisection method less than rate of Newton's Method: $1/2 < 1 - 1/3$, so we choose Bisection method.

7. Exercise 1.5.1.

8. Exercise 1.5.2.

   (a) $c_1 = 1.6$ and $c_2 = 1.742268$.

   (b) $c_1 = 1.57877072$ and $c_2 = 1.66016001$.

   (c) $c_1 = 1.09290658$ and $c_2 = 1.11935668$.