

**Midterm Exam 1. (Practice)**

Math 353 Section 12, Fall 2008, University of Delaware

1. Convert the following binary numbers to decimal form.

(a)  $(10101.\overline{0011})_2$  (b)  $(111000.11011)_2$

2. Convert the following numbers to binary form and find its  $fl(x)$ .

(a) 87.1 (b)  $\frac{1}{7}$  (c)  $\frac{75}{128}$

3. Derive the second-order centered difference formula for the third derivative

$$f'''(x) \approx \frac{f(x+2h) - 2f(x+h) + 2f(x-h) - f(x-2h)}{2h^3}$$

and find its the truncation error term  $E(f, h)$ .

4. Use the Richardson's extrapolation to derive the following formula.

$$f'(x) \approx \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h}$$

and determine its order.

5. Let  $f(x) = \cos x$ . Use the centered three-point difference formula for the second derivative to approximate  $f''(\pi/4)$ , for  $h = 0.1, 0.01, 0.001$ . Determine the optimal step size  $h_{optimal}$ .

6. Let consider the following equations

(a)  $f(x) = 1 - x + \cos x = 0$  (b)  $f(x) = e^x - 2 - x = 0$  (c)  $f(x) = x - 5 + \ln x$

- (a) Use the Intermediate Value Theorem to find an interval of length one that contains a root of the equations.  
(b) Apply two steps of the Bisection Method for the each equations.  
(c) Apply two steps of the Regula-Falsi Method for the each equations.  
(d) How many steps of the Bisection Method are required to find the solution within  $10^{-7}$ ?
7. Let consider  $f(x) = x^3 + 4x^2 - 10 = 0$ .

- (a) Manipulate the equation to the Fixed-Point problem at least 3 different ways.  
(b) For each case determine whether Fixed-Point Iteration is convergent.
8. Apply two steps of the Secant Method to the equation  $f(x) = x^2 - 2x - 1$  with initial guesses  $x_0 = 2.6$  and  $x_1 = 2.5$ .
9. Let consider  $f(x) = xe^{-x}$ .

- (a) Find the Newton's Method formula.  
(b) If  $x_0 = 0.2$ , then find  $x_1, x_2$  and  $x_3$ . What is  $\lim_{n \rightarrow \infty} x_n$ ?  
(c) If  $x_0 = 20$ , then find  $x_1, x_2$  and  $x_3$ . What is  $\lim_{n \rightarrow \infty} x_n$ ?

10. Can Newton's Method be used to solve  $f(x) = \sqrt[3]{x}$ ? Why?
11. Solve the following system by Gaussian Elimination (a) naive (b) with partial pivoting.
- $$\begin{aligned}x_1 - 2x_2 + 7x_3 &= -4 \\4x_1 + 2x_2 + x_3 &= -6 \\2x_1 + 5x_2 - 2x_3 &= 1\end{aligned}$$
12. Solve the system  $AX = B$  by finding the  $LU$  factorization for (a)  $B = (8, -4, 10, -4)^\top$  (b)  $B = (28, 13, 23, 4)^\top$ , where

$$A = \begin{pmatrix} 4 & 8 & 4 & 0 \\ 1 & 5 & 4 & -3 \\ 1 & 4 & 7 & 2 \\ 1 & 3 & 0 & -2 \end{pmatrix}$$

What is the condition number of  $A$  ?

13. Solve the system by finding the  $PA = LU$  factorization.
- $$\begin{aligned}x_1 + 8x_2 + x_3 &= -4 \\4x_1 + 8x_2 - 0.5x_3 &= -6 \\4x_1 + 17x_2 + 4.75x_3 &= -9\end{aligned}$$
14. Exercises 2.5.1 - 2.5.4.
15. Exercises 2.7.1 and 2.7.4 - 2.7.5.