Name: Math 241: Calculus & Analytic Geometry A
Section:

Final Exam
10 December 2007

Instructions: **Show all work to receive full or partial credit.** All University rules and guidelines for student conduct are applicable.

The science of mathematics presents the most brilliant example of how pure reason may successfully enlarge its domain without the aid of experience. –Emmanuel Kant.

Question	Score
1	
2	
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Total	

1. Find the value(s) of k that make $f(x)$ continuous everywhere, where

$$f(x) = \begin{cases} \frac{x^2-4}{x-2}, & \text{if } x < 2 \\ kx + 8, & \text{if } x \geq 2. \end{cases}$$

2. Differentiate

$$f(x) = e^x(x^2 + \tan x).$$

3. Calculate $\frac{dy}{dx}$ where

$$y = \frac{x + \cos x}{1 + e^x}.$$

4. Differentiate the function,

$$f(x) = \csc^2\left(x^2 + \frac{1}{x}\right).$$

5. Find the value of $\frac{dy}{dx}$ at the point $x = 0$ and $y = 0$ where x and y are related by the following equation:

$$x \cos y = y \cos x.$$

6. A spherical iron ball 8 inches in diameter is coated with a layer of ice of uniform thickness. If the ice melts at the rate of 10 in^3/min, how fast is the thickness of the ice decreasing when it is 2 inches thick?

7. Using linearization or differentials, estimate

$$\ln(1.01).$$

8. A bacteria culture initially contains 50 cells and grows at a rate proportional to its size. After an hour the population has increased to 100.

   (a) Find an expression for the number of bacteria after t hours.

   (b) When will the population reach 800?

9. Prove that the function

$$f(x) = x^3 - x^2 + 8$$

has at least one root.

10. Consider

$$y = \frac{x}{x^2 - 9}.$$

Use the space below to neatly organize your work, and record your answer on the lines/axes provided.

(a) Domain: _____

(b) x- and y- intercepts: _____

(c) Asymptotes (vertical and horizontal): _____

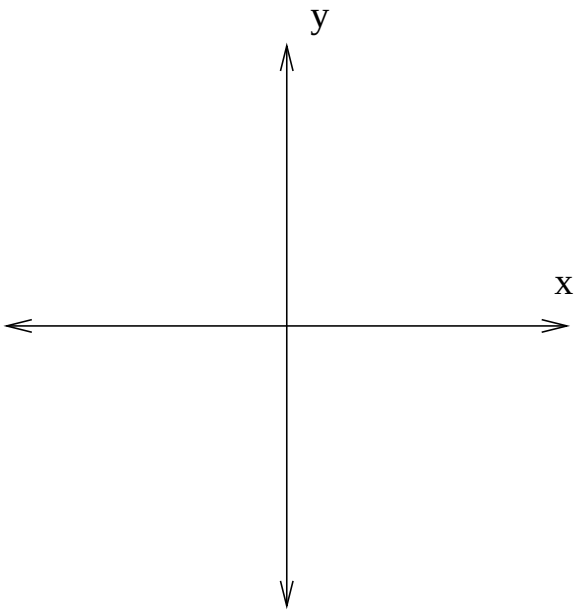
(d) Intervals of increase/decrease: _____

(e) Local min/min: _____

(f) Intervals of concavity: _____

(g) Inflection point(s) (if any): _____

(h) Sketch:



11. Find the absolute maximum and minimum values of the $f(x)$,

$$f(x) = (x^2 + x)^3$$

on the interval $-2 \leq x \leq 1$.

12. Find the point P on the curve defined by $y = \sqrt{x}$ that is nearest the point Q(4, 0).

13. Find the most general antiderivative of the function

$$f(x) = e^x + \sin(x).$$

14. Find the derivative of the function

$$f(x) = \int_0^{\sin x} \frac{\sqrt{1-t^2}}{\cos t} dt$$

15. Evaluate

$$\int_{-\pi}^{\pi} (\cos(x) + \cos(x/2) + \cos(2x))dx.$$

16. Evaluate the indefinite integral.

$$\int \frac{x}{\sqrt[4]{x^2+4}}dx$$