Preliminary Review.
Math 241 Sections 20-22, Fall 2008, University of Delaware

Basic Rules of Inequalities:

1. If $a \le b$, then $a + c \le b + c$ for any real number c.
2. If $a \le b$ and $c \le d$, then $a + c \le b + d$.
3. If $a \le b$ and $c > 0$ ($c < 0$), then $ac \le bc$ ($ac \ge bc$).
4. If $0 < a \le b$, then $1/a \ge 1/b$.

Intervals: Let a and b be any real numbers.
$(a, b) = \{x \in \Re \,|a < x < b\}$ The open interval.
$[a, b] = \{x \in \Re \,|a \le x \le b\}$ The closed interval.
$[a, b) = \{x \in \Re \,|a \le x < b\}$ and $(a, b] = \{x \in \Re |a < x \le b\}$ The half open intervals.
$(a, \infty) = \{x \in \Re \,|a < x\}$, $[a, \infty) = \{x \in \Re \,|a \le x\}$, $(-\infty, a) = \{x \in \Re \,|x < a\}$ and
$(-\infty, a] = \{x \in \Re \,|x \le a\}$ The infinite intervals.
$(-\infty, \infty) = \Re$ Set of all real numbers.

Absolute value and its properties: Absolute value of a number a, denoted by $|a|$, defined as follows:

$$|a| = \begin{cases} a, & \text{if } a \ge 0 \\ -a, & \text{if } a < 0. \end{cases}$$

Let a and b be any real numbers.

1. $|ab| = |a||b|$.
2. $|a/b| = |a|/|b|$, where $b \ne 0$.
3. Solution of the equation $|x| = a$ is $x = \mp a$, where $a > 0$
4. Solution of the inequality $|x| < a$ is $(-a, a)$, where $a > 0$
5. Solution of the inequality $|x| > a$ is $(-\infty, -a) \cup (a, \infty)$, where $a > 0$
6. $|a + b| \le |a| + |b|$.

The distance between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in Cartesian coordinate system is

$$|P_1P_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

The slope of a line (non vertical) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

where $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are arbitrary points in the line.
Equations of the line:
Point-Point. An equation of the line passing through the given points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

Point-Slope. An equation of the line with slope m and passing through the point $P_1(x_1, y_1)$ is

$$y - y_1 = m(x - x_1).$$

Slope-Intercept. An equation of the line with slope m and y axis intercept b is

$$y = mx + b.$$

General form of the equation of a line is

$$Ax + By + C = 0.$$

Let we have two lines $y = m_1x + b_1$ and $y = m_2x + b_2$.
1. They are parallel if and only if $m_1 = m_2$.
2. They are perpendicular if and only if $m_1 = -1/m_2$.

Function:
A function is a rule that assigns a unique real number to each number in a specified set of real numbers.
A graph of a function f is the set of points in coordinate system with coordinations $\{(x, f(x))\,|x$ in domain of the $f\}$.
A function f is said to be ***increasing*** (***decreasing***) on an interval **I**, if for all numbers $x_1 < x_2$ in interval **I** following is true

$$f(x_1) < f(x_2) \;\; (f(x_1) > f(x_2)).$$

If $f(-x) = f(x)$ $(f(-x) = -f(x))$ true for all x in domain, then f is called ***even*** (***odd***) function.
If a function f defined by different formulas in different parts of its domain, then f is called ***piecewise*** defined function.

Algebra of functions: Let we have two functions f and g with domains A and B.

1. $(f \mp g)(x) = f(x) \mp g(x)$, domain: $A \bigcap B$.
2. $(fg)(x) = f(x)g(x)$, domain: $A \bigcap B$.
3. $(f/g)(x) = f(x)/g(x)$, domain: $A \bigcap B$ and $g(x) \neq 0$..
4. $(f \circ g)(x) = f(g(x))$, is called composition of f and g.

If $f(x) = x^a$, where a is a constant, then f is called a ***power*** function.
If $f(x) = a_nx^n + a_{n-1}x^{n-1} + ... + a_1x + a_0$, where $a_0,\ a_1, ..., a_n$ are constants and n is positive integer, then f is called a ***polynomial*** with degree n.
If $f(x) = P(x)/R(x)$, where $P(x)$ and $R(x)$ are polynomials, then f is called a ***rational*** function.

Trigonometry: Angles measured in degrees or radians.

$$\pi \text{ rad} = 180^\circ.$$

Trigonometric functions. Let $P(x,y)$ be any point on the terminal side of a angle θ located in coordination system in standard position and we let $r = |OP|$. Then we define

$$\sin\theta = \frac{y}{r}, \quad \cos\theta = \frac{x}{r}, \quad \tan\theta = \frac{y}{x}$$
$$\csc\theta = \frac{r}{y}, \quad \sec\theta = \frac{r}{x}, \quad \cot\theta = \frac{x}{y}.$$

Basic Identities:

1. $\csc\theta = \frac{1}{\sin\theta}$, $\sec\theta = \frac{1}{\cos\theta}$, $\cot\theta = \frac{1}{\tan\theta}$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$.
2. $\sin^2\theta + \cos^2\theta = 1$.
3. $\cos(-\theta) = \cos(\theta)$, even $\sin(-\theta) = -\sin(\theta)$, $\tan(-\theta) = -\tan(\theta)$ odd.
4. $\cos(\theta + 2\pi) = \cos(\theta)$, $\sin(\theta + 2\pi) = \sin(\theta)$ periodic.
5. $\sin 2\theta = 2\sin\theta\cos\theta$, $\cos 2\theta = \cos^2\theta - \sin^2\theta$.
6. $\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$, $\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$.

Exponential functions and Logarithms:
If $f(x) = a^x$, where a is positive constant, then f is called ***exponential*** function. Domain: set of all real numbers. Range: set of all positive real numbers.
Laws of Exponents: Let a and b are given positive real numbers and x and y are any real numbers. Then

1. $a^x a^y = a^{x+y}$, $\frac{a^x}{a^y} = a^{x-y}$.
2. $(ab)^x = a^x b^x$.
3. $(a^x)^y = a^{xy}$.

Let we have a one-to-one function f with domain A and range B. Then its inverse function is defined by

$$f^{-1}(y) = x \iff f(x) = y$$

and it has domain B and range A.
If f is inverse function of the exponential function, then $f(x) = \log_a x$ is called ***logarithmic*** function with base a;

$$\log_a x = y \iff a^y = x.$$

Domain: Set of all positive numbers. Range: set of all real numbers.
Properties and Laws of Logarithms:

1. $\log_a(a^x) = x$, for all real numbers x.
2. $a^{\log_a x} = x$, for all positive real numbers x.
3. $\log_a(xy) = \log_a x + \log_a y$, $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$, where x and y are positive numbers.
4. $\log_a(x^b) = b\log_a x$, where x is positive and b is any real number.
5. $\log_a x = \frac{\log_b x}{\log_b a}$, where a, b and x are positive numbers.

The logarithm with base $e = 2.71828...$ is called natural logarithm;

$$\log_e x = \ln x.$$