Lecture Summary 3.
Math241 Sections 20-22, Fall 2008 University of Delaware

Maximum and Minimum (absolute). If there is a number b in domain of a function f such that $f(x) \leq f(b)$ (or $f(x) \geq f(b)$) holds true for all x in domain of f, then we say f has an absolute max (or an absolute min) at b. The number $f(b)$ is called an absolute max (or min) value of f on the its domain.
Maximum and Minimum (local). If there is an open interval containing b such that $f(x) \leq f(b)$ ($f(x) \geq f(b)$) holds true for all x in the interval and the domain, then b is called local max (local min) of f.
Extremum Points: All maximum or minimum (absolute or local) points are called extremum points of f.
Fermat's Theorem. If b is an extremum point of f and $f'(b)$ exists, then $f'(b) = 0$.
Critical Points. If for some point b in domain of f: either $f'(b) = 0$ or $f'(0)$ does not exist, then b is called a critical point of f.
Theorem. If b is an extremum point of f, then b is a critical point of f.
The first derivative test. Let b be a critical point of continuously differentiable function f. Then
1) If f' changes its sign from positive to negative at b, then b is a local max point of f.
2) If f' changes its sign from negative to positive at b, then b is a local min point of f.
3) If f' does not change its sign at b, then b is not extremum point of f.
The second derivative test. Let $f''(x)$ is continuous near c. Then
1) If $f'(c) = 0$ and $f''(c) < 0$, then c is a max (local) of f
2) If $f'(c) = 0$ and $f''(c) > 0$, then c is a min (local) of f
3) If $f'(c) = 0$ and $f''(c) = 0$, then this test is inconclusive.
Absolute value test. Let c be a critical point of a continuously differentiable function f defined on an interval $\mathbf{I}$.
1) If $f'(x) < 0$ for all $x < c$ in $\mathbf{I}$ and $f'(x) > 0$ for all $x > c$ in $\mathbf{I}$, then c is absolute min point of f on $\mathbf{I}$
2) If $f'(x) > 0$ for all $x < c$ in $\mathbf{I}$ and $f'(x) < 0$ for all $x > c$ in $\mathbf{I}$, then c is absolute max point of f on $\mathbf{I}$

The Extreme Value Theorem. If f is continuous on a closed interval $[a, b]$, then f attains its absolute max and min values in $[a, b]$.
The Closed Interval Method. Let f is continuous on a closed interval $[a, b]$. To find absolute max and min value of f on $[a, b]$, we do the following 3 steps procedure:

1. Find the values of f at the all critical points of f in (a, b).
2. Compute $f(a)$ and $f(b)$.
3. The largest value of Step 1 and 2 is absolute max value. The smallest value of Step 1 and 2 is absolute min value.

Rolle's Theorem. If f satisfies the following 3 conditions:
a) f is continuous on $[a, b]$
b) f is differentiable on (a, b)
c) $f(a) = f(b)$,
then there is a number c in (a, b) such that $f'(c) = 0$.

The Mean Value Theorem. If f is continuous on $[a, b]$ and differentiable on (a, b), then there is a number c in (a, b) such that

$$f(b) - f(a) = f'(c)(b - a).$$

Theorem. If $f'(x) = 0$ for all x in (a, b), then f is constant function on (a, b).
Corollary. If $f'(x) = g'(x)$ for all x in an interval $\mathbf{I}$, then $f(x) = g(x) + C$ on $\mathbf{I}$, where C is a constant.

Intervals of Increase/Decrease.
a) If $f'(x) < 0$ on an interval, then f is decreasing on that interval
b) If $f'(x) > 0$ on an interval, then f is increasing on that interval
Concavity. If the graph of f lies above all of its tangents on an interval, then it is called concave up on that interval. If the graph of f lies below all of its tangents on an interval, then it is called concave down on that interval.
Concavity Test.
a) $f''(x) < 0$ on an interval, then f is concave down on that interval
b) $f''(x) > 0$ on an interval, then f is concave up on that interval.
Inflection point. If f is continuous at b and it changes its concavity at b, then b is called an inflection point of f.

Antiderivative. If $F'(x) = f(x)$ for all x in an interval $\mathbf{I}$, then f is called an antiderivative of f on $\mathbf{I}$.
Theorem. If F_1 is particular antiderivative of f on an interval $\mathbf{I}$, then the most general antiderivative of f on $\mathbf{I}$ is $F(x) = F_1(x) + C$, where C is arbitrary constant.

Optimization Problem (Examples).
Example 1. Find the dimensions of the rectangle with perimeter 100 m, whose area is as large as possible.
Example 2. Find the point(s) on the hyperbola $y^2 = 4 + x^2$ that is closest to the point $P(2, 0)$.
Example 3. Cylindrical can is made to contain $V\ cm^3$ of liquid. Find the dimensions that will minimize the cost of the metal to make the can.
Example 4. A steel pipe is being carried down a hallway 9 ft wide. At the end of the hall there is a right-angled turn into a narrower hallway 6 ft wide. What is the length of the longest pipe that can be carried horizontally around the corner.