Midterm Exam 3. (Practice)
Math 241 Sections 20-22, Fall 2008 University of Delaware

1. Find the extremum point(s) of the function.
   (a) $f(x) = \sqrt[3]{(x^2-4)^2}$
   (b) $f(x) = (x^3 - 2x)^2$
   (c) $f(x) = \ln x^2 - x^2$
   (d) $f(x) = e^{x^3-3x+1}$

2. Find the absolute max and min values of f on the given interval.
   (a) $f(x) = 2x - 3\sqrt[3]{x^2}, \quad [-1, 3]$
   (b) $f(x) = 2\sin x - \cos 2x, \quad [0, 2\pi]$

3. A wire 36 *cm* in length is cut into two parts, one of which has length x, $1 \le x \le 35$. The both pieces are bent into form of a square. How should the wire be cut

   a) To maximize total area enclosed?

   b) To minimize total area enclosed?

4. A rectangular area of 160000 square feet must be enclosed. How should this be done in order to use the minimum amount of fencing?

5. Find the intervals of increase/decrese and intervals of concavity of f.
   (a) $f(x) = x\sqrt{2+x}$
   (b) $f(x) = \sin^2 x - 2\cos x$
   (c) $f(x) = \ln(x^2 + 1)$

6. Find $f(x)$, if
$$f'(x) = 2 - x^2 + 2\sin x \text{ and } f(0) = 1.$$

7. Find $f(x)$, if
$$f''(x) = 2 - 12x, \quad f'(0) = 9 \text{ and } f(2) = 15.$$

8. Show that the equation $3x + 2\cos x + 5 = 0$ has exactly one solution.

9. By applying the Mean Value Theorem to the function $f(x) = \sqrt[5]{x}$ on the interval $[32, 33]$, show that
$$\sqrt[5]{33} < 2.0125$$

10. Evaluate following integrals by interpreting each in terms of areas
    (a) $\int_0^6 (\frac{2}{3}x - 2)\, dx$
    (b) $\int_{-1}^1 (1 - x)\, dx$

11. Find $\int_{-1}^1 f(x)\, dx$, if
$$f(x) = \begin{cases} 1 - x^2, & \text{if } -1 \le x < 0 \\ \sqrt{1-x^2}, & \text{if } 0 \le x \le 1 \end{cases}.$$

12. Evaluate
$$\int_0^4 (2\sqrt{x} - x^2 + 3)\, dx.$$

Answers.

1. (a) $x = \mp 2 \ - \ min \ (global), \quad x = 0 \ - max$
   (b) $x = \mp\sqrt{2}, \ 0 \ - \ min, \quad x = \mp\sqrt{2/3} \ - \ max$
   (c) $x = \mp 1 \ - \ max \ (global)$
   (d) $x = -1 \ - max \quad x = 1 \ - min$
2. (a) $f(0) = 0 \ - \ Absolute\ max, \quad f(-1) = -5 \ - \ Absolute\ min$
   (b) $f(\pi/2) = 3 \ - \ Absolute\ max, \quad f(7\pi/6) = f(11\pi/6) = -1.5 \ - \ Absolute\ min$
3. (a) $x = 1$ or $x = 35$
   (b) $x = 18$
4. Length and width of the rectangle are equal to 400 we will use the minimum amount of fencing which is $1600 feet$.
5. (a) Increase on $(-4/3, \infty)$ and decrease on $(-2, -4/3)$. Concave up on $(-2, \infty)$.
   (b) Increase on $(2\pi k, 2\pi(2k+1))$ and decrease on $(2\pi(2k-1), 2\pi k)$, where k is integer. Concave up on $(2\pi k - \pi/3, 2\pi k + \pi/3)$ and concave down on $(2\pi k + \pi/3, 2\pi k + 5\pi/3)$.
   (c) Increase on$(-\infty, 0)$ and decrease on $(0, \infty)$. Concave up on $(-1, 1)$ and concave down on $(-\infty, -1) \cup (1, \infty)$.
6. $f(x) = 2x - x^3/3 - 2\cos x + 3$
7. $f(x) = -2x^3 + x^2 + 9x + 9$
8.
9.
10. (a) 0
    (b) 2
11. $2/3 + \pi/4$
12. $4/3$