

Midterm Exam 1. (Practice)

Math 241 Sections 20-22, Fall 2008. University of Delaware.

1. Find the exact value of each expression.

a) $e^{2 \ln \sqrt{3}}$ b) $\log_{10} 25 + \frac{1}{2} \log_{10} 64 - \log_{10} 2$ c) $\cos(\arctan(2\sqrt{2}))$

2. Solve the equation.

a) $2 \ln x - \ln(x + 1) = 1$ b) $3^{2x+1} = 6.$

3. Find a formula for the inverse of the given function.

a) $f(x) = \sqrt{2x - 1}$ b) $f(x) = \frac{\ln x}{\ln x + 1}$

4. Evaluate the following limits, if they exist.

a) $\lim_{x \rightarrow 4} \frac{\sqrt{x+5} - 3}{x-4}$ b) $\lim_{x \rightarrow 0.5} \left(\sqrt[3]{2x-1} + \frac{x^2-1}{x^2+x+1} \right)$
c) $\lim_{x \rightarrow 0} (\sqrt{x+1} + |x|)$ d) $\lim_{x \rightarrow 0} (e^x - 3 \cos x + \sin^3 x)$

5. Find the vertical asymptotes of the function.

a) $f(x) = \frac{3x+1}{x^2+x-12}$ b) $f(x) = \frac{1}{\ln x}$

6. Find the limit

$$\lim_{x \rightarrow 0} \sqrt{x^2 + x^4} \left(1 - \cos^2 \frac{\pi}{x^2} \right)$$

if it exists.

7. For what value of c is $f(x)$ continuous on $(-\infty, +\infty)$?

$$f(x) = \begin{cases} cx + e^{3-x}, & \text{if } 3 \geq x \\ cx^2 - 1, & \text{if } 3 < x \end{cases}$$

8. Use continuity to evaluate the limit.

$$\lim_{x \rightarrow 2} \left(\ln(x^2 - 3) + \arcsin \left[\frac{x^3 - 2x - 3}{e^{x-2}} \right] \right)$$

9. Find an equation of the tangent line to the curve

$$y = \frac{x-1}{x-2} \text{ at } P(3, 2).$$

10. Show that the equation $\tan x + x^2 = 1 - x^2$ has at least one real root.

11. Using the definition of derivative, find $f'(1)$ if

$$f(x) = \frac{1}{\sqrt{x+1}}.$$

12. Find the horizontal asymptotes of the function.

a) $f(x) = \frac{\sqrt{x^2+x+1}}{x-2}$ b) $f(x) = \sqrt{x^2+4x} - \sqrt{x^2+x}$

Answers.

1. a) 3 b) 2 c) $\frac{1}{3}$

2. a) $x_{1,2} = \frac{e \mp \sqrt{e^2 + 4e}}{2}$ b) $x = \log_3 \sqrt{2}$

3. a) $f^{-1}(x) = \frac{x^2 + 1}{2}$ b) $f^{-1}(x) = e^{\frac{x}{1-x}}$

4. a) $\frac{1}{6}$ b) $-\frac{3}{7}$ c) 1 d) -2

5. a) $x = -4$ and $x = 3$ b) $x = 1$

6. 0, *Hint : Use The Squeeze Theorem.*

7. $c = \frac{1}{3}$

8. $\frac{\pi}{2}$

9. $y = -x + 5$

10. *Hint : Use The Intermediate Value Theorem.*

11. $f'(1) = -\frac{1}{2\sqrt{8}}$

12. a) $y = \mp 1$ b) $y = \mp \frac{3}{2}$