IMPROVING THE RATE OF CONVERGENCE OF ‘HIGH ORDER FINITE ELEMENTS’ ON POLYHEDRA II: MESH REFINEMENTS AND INTERPOLATION

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Abstract. We construct a sequence of meshes $T_k'$ that provides quasi-optimal rates of convergence for the solution of the Poisson equation on a bounded polyhedral domain with right hand side in $H^{m-1}$, $m \geq 2$. More precisely, let $\Omega \subset \mathbb{R}^3$ be a bounded polyhedral domain and let $u \in H^1(\Omega)$ be the solution of the Poisson problem $-\Delta u = f \in H^{m-1}(\Omega)$, $m \geq 2$, $u = 0$ on $\partial \Omega$. Also, let $S_k$ be the Finite Element space of continuous, piecewise polynomials of degree $m \geq 2$ on $T_k'$ and let $u_k \in S_k$ be the finite element approximation of $u$, then $\|u - u_k\|_{H^1(\Omega)} \leq C \dim(S_k)^{-m/3} \|f\|_{H^{m-1}(\Omega)}$, with $C$ independent of $k$ and $f$. Our method relies on the a priori estimate $\|u\|_D \leq C \|f\|_{H^{m-1}(\Omega)}$ in certain anisotropic weighted Sobolev spaces $D = \mathcal{D}^{m+1}_a(\Omega)$, with $a > 0$ small, determined only by $\Omega$. The weight is the distance to the set of singular boundary points (i.e., edges). The main feature of our mesh refinement is that a segment $AB$ in $T_k'$ will be divided into two segments $AC$ and $CB$ in $T_{k+1}'$ as follows: $|AC| = |CB|$ if $A$ and $B$ are equally singular and $|AC| = \kappa |AB|$ if $A$ is more singular than $B$. We can choose $\kappa \leq 2^{-m/a}$. This allows us to use a uniform refinement of the tetrahedra that are away from the edges to construct $T_k$.

Contents

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C.B. was supported by NSF Grant DMS-0713125 and by University of Delaware Research Foundation. V.N. was supported by NSF Grant DMS 0555831. L.T.Z. was supported by a NSF grants DMS-058110, DMS-0619587, and by Lawrence Livermore National Laboratory under subcontract number B551021.