1 Course Description:

This course introduces the finite element method (FEM) and illustrate its applicability to solving Partial Differential Equations (PDEs) by algorithm implementation and numerical experiments in MATLAB. The focus is on PDES that arise from modeling incompressible fluid flow. Topics include advanced iterative multilevel methods, computational aspects of the FEM method, discretization of saddle point problems, and applications.

2 Course Objectives:

• Students will apply the finite element method to solving Partial Differential Equations (PDEs) by algorithm implementation and numerical experiments in MATLAB.

• Students will write their own MATLAB programs and use them to design new algorithms.

• Students will choose appropriate advanced iterative multilevel methods to solve saddle point problems and related PDEs.

• Students will combine advanced calculus and linear algebra knowledge with scientific computing skills to solve concrete fluid flow models.

• Students will conduct numerical experiments in MATLAB.

• Students will write scientific reports that incorporate the design of the algorithms and programs, the mathematical justification of the design, the numerical results and their interpretation in terms of the given problem.

• Students will reflect on how to apply the finite element method in other situations or contexts.
3 Course Outline:

1. The Laplace equation in 2D
   - Weak solutions and variational formulation of elliptic PDE’s
   - Introduction to the finite element method.
   - Interpolation theory
   - The Ritz-Galerkin method
   - Approximation properties and error bounds

2. Numerical implementation of the method
   - Finite element spaces.
   - Discrete variational formulation
   - Writing the FEM linear system
   - Preconditioning. The conjugate gradient method
   - Numerical implementation and experiments with MATLAB

3. Saddle point problems
   - Existence and uniqueness of solutions
   - Approximation of the problem
   - Exact and inexact Uzawa’s algorithms
   - Multilevel Algorithms

4. Stokes systems
   - Weak formulation
   - Discretization by Inexact Uzawa method
   - Numerical implementation and experiments with MATLAB

5. Navier-Stokes systems
   - Weak formulation and linearization
   - Discretization by non-symmetric Uzawa method
   - Numerical implementation and experiments with MATLAB

4 Course credit:

3 credit hours
5 Prerequisite:

- Multi-variable Calculus
- Linear algebra (Math 342 or Math 349)
- Basic knowledge of programming in MATLAB (or Math 353)

6 Assessment:

Programming assignments = 30%, Projects = 30%, Reflection Papers = 10%, Final Scientific Report = 30%.

7 Textbook:

The material to be covered will be selected from the following references.

References


