University of Delaware
Department of Mathematical Sciences
Math 353 Engineering Mathematics III
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Homework 8

Consider the differential equation

$$y' = t^2 - y,$$

with the general solution: $y(t) = Ce^{-t} + t^2 - 2t + 2$.

1) Plot the vector field (at least 100 vectors) of the function $f(t, y) = t^2 - y$ on the rectangle $S = \{(t, y)|0 \leq t \leq 4, 0 \leq y \leq 10\}$ and on the same system of coordinates plot the solutions for $C = -2, -1, 1, 2$. Comment the plot.

2) Find the exact solution of The Initial Value Problem (IVP)

$$y' = t^2 - y, \; y(0) = 1, \; t \in [0, 1]. \tag{1}$$

3) Let $h = 0.2$ and do two steps of **Euler's Method** by hand calculation for the IVP of problem 2). Check the result with the function **euler.m** from the class webpage.

4) Let $h = 0.2$ and do two steps of **Explicit Trapezoid or Heun's Method** by hand calculation for the IVP of problem 2). Check the result with the function **heunT.m** from the class webpage.

5) Let $h = 0.2$ and do one step of **Runge-Kutta Method of order four** by hand calculation for the IVP of problem 2). Check the result with the function **rk4.m** from the class webpage.

6) A resonant spring system with periodic forcing function is modeled by

$$x''(t) + 25x(t) = 8\sin(5t) \quad with \; x(0) = 0 \;\; and \;\; x'(0) = 0.$$

Write the ODE and the IC's as a first order system, and then solve the corresponding system using Runge-Kutta method order four over the interval [0 2] and with the step size $h = 0.05$. **Hint:** You can use the functions from class webpage, e.g., **rk4s.m**.

7) *Predator-prey model.* Let $x(t)$, and $y(t)$ denote the population of rabbits and foxes, at time t. The predator-prey model asserts that $x(t)$, and $y(t)$ satisfy

$$x'(t) = Ax(t) - Bx(t)y(t),$$
$$y'(t) = Cx(t)y(t) - Dy(t).$$

Take $A = 2, B = 0.02, C = 0.0002, D = 0.8$, and use Runge-Kutta method order four over the interval [0 5] and with the step size $h = 0.1$ to approximate the solution of the above system if

(a) $x(0) = 3000$ rabbits and $y(0) = 120$ foxes

(b) $x(0) = 5000$ rabbits and $y(0) = 100$ foxes

Write your own Runge Kutta function or apply one of the functions from class webpage. Plot the solution and comment the final results.

Note: Hand in copies of the MATLAB commands, outputs involved in the above problems and **ONLY the functions you create**. Comment the results. **HUGE deduction will be considered for printing unnecessary pages.**

Extra Credit 20p: The (IVP),

$$y'' + y + y^3 = 0, \quad 0 \le t \le 4\pi, \; y(0) = \alpha, \; y'(0) = 0,$$

models a mass-spring-damper system with a hard spring.

a) Write a MATLAB code or use a book function or a code from the class web page to solve the above (IVP) for $\alpha = .1, 1, 2$. Use $h = \frac{\pi}{12}$.

b) Solve the (IVP) for $\alpha = .1, 1, 2$ by using **ode45** instead. The MATLAB function **ode45** is based on R-K method of order 6 and adapts the step size. To learn how to use the OPTIONS of this function check **help ode45** and see the script file *ode_ex.m* from class web page. Use an absolute value tolerance of 10^{-8} and a relative tolerance of $10^{-}7$. For $\alpha = 2$ plot the two numerical solutions (your code vs ode45) on the same system of coordinates. Comment the results.

c) Multiply the Differential Equation from the above IVP by y' and integrate on $[0, t]$, ($t \in [0, 4\pi]$), to get the energy equation

$$2\left(y'(t)\right)^2 + 2y^2(t) + y^4(t) = 2\alpha^2 + \alpha^4.$$

Show all the steps.

d) Make a plot of the difference between the computed value of this energy and the constant vector with components $2\alpha^2 + \alpha^4$ for each α. Compare the two methods. **Comment on how well each method keeps the energy constant.**

Attach all script and function files created by you. Provide a diary file with the MATLAB commands and outputs needed for the extra credit part.