GUIDELINES FOR THE METHOD OF UNDETERMINED COEFFICIENTS

Given the *constant coefficient* linear differential equation

$$a\,\ddot{x} + b\,\dot{x} + c\,x = f(t),$$

where $f(t)$ is an exponential, a simple sinusoidal function, a polynomial, or a product of these functions:

1. Solve the homogeneous equation for a pair of linearly independent solutions $x_1(t)$ and $x_2(t)$.

2. If $f(t)$ is *not* a solution of the homogeneous equation, take a trial solution of the same type as $f(t)$ according to the suggestions given in class.

3. If $f(t)$ is a solution of the homogeneous equation, take a trial solution of the same type as $f(t)$ multiplied by the lowest power of t for which no term of the trial solution is a solution of the homogeneous equation.

4. Substitute the trial solution into the differential equation and solve for the undetermined coefficients so that it is a particular solution $x_p(t)$.

5. Set $x(t) = x_p(t) + c_1x_1(t) + c_2x_2(t)$, where the constants c_1 and c_2 can be determined if initial conditions are given.

6. If $f(t)$ is a sum of forcing functions of the type described above, split the problem into simpler parts. Find a particular solution for each part, then add the particular solutions to obtain $x_p(t)$.