SAMPLE: M243 Examination II

THERE IS NO GUARANTEE THAT THE COVERAGE ON THE ACTUAL HOUR EXAM WILL COINCIDE WITH THE PROBLEMS IN THIS SAMPLE OR THAT THERE WILL BE THE SAME NUMBER OR TYPE OF QUESTIONS.

1. (20 points) Do the following:
   
   (a) For \( f(x, y) = x\sqrt{2y^2 + x}\sin(y^2) \) find \( \frac{\partial f}{\partial x} \) and \( \frac{\partial f}{\partial y} \).

   (b) For \( g(x, y) = x^2 - y^2 \), find \( \frac{\partial^2 g}{\partial x^2} \) and \( \frac{\partial^2 g}{\partial y^2} \).

2. (25 points) Given the function \( P(x, y) = \ln\left(\frac{1}{\sqrt{x^2 + y^2}}\right) \), find:

\[
\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2}
\]

3. (25 points) The position of a particle moving in \( \mathbb{R}^3 \) at time \( t \geq 0 \) is given by

\[
2t\hat{i} + \sin(t^2)\hat{j} - t^2\hat{k}.
\]

Find the unit tangent vector and the unit normal to the curve. In which direction if the normal pointing with respect to the center of curvature at \( t = 1 \)?

4. (30 points)

Let \( r(t) \) denote the position in \( \mathbb{R}^2 \) of a moving object at time \( t \).

   (a) If \( r(t) = (x(t), y(t))^\top \), what is \( ||r|| \)?

   (b) Given the function \( p(x, y) = \frac{1}{||r||} \), find the gradient of \( p \). (\textbf{HINT:} Look at the information given in problem #2 above!)

   (c) Let \( r(t) = \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix} \). Find the unit tangent vector \( T(\theta) \) and unit normal \( N(\theta) \) to the curve.

   (d) Find the dot product \( T \cdot \nabla(p) \) where \( p \) is the function of part (b). What does this result say about these vectors \( N \) and \( \nabla(p) \)?

\textbf{HINT:} Write \( \nabla(p) \) in terms of polar coordinates with \( r := ||r|| \).