SAMPLE APPLIED PROBLEMS
for
CHAPTER 2

(1) A body of mass 5 slugs [1] dropped from a height of 100 feet with zero initial velocity. Assuming no air resistance, find

(a) an expression for the velocity of the body at time t;
(b) an expression for the position of the body at any time t;
(c) the time required for the body to reach the ground.

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(2) A steel ball weighing 2 pounds is dropped from a height of 3000 feet with no initial velocity. As it falls, the ball encounters air resistance numerically equal to $(v/8)$ ft./pound-second. Find

(a) the limiting velocity of the ball in feet/second;
(b) the time required for the ball to hit the ground.

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(3) A body of mass m is thrown vertically into the air with an initial velocity v_0. If the body encounters air resistance proportional to its velocity, find

(a) the equation of motion;
(b) an expression for the velocity of the body at any time t;
(c) the time at which the body reaches its maximum height.

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(4) A college dormitory houses 100 students, each of whom is susceptible to a certain virus infection. A simple model of epidemics assumes that during the course of an epidemic the rate of change with respect to time of the number of infected students I is proportional to the number of infected students and is also proportional to the number of uninfected students.

(a) If at time $t = 0$ a single student becomes infected, show that the number of infected students at time t is give by

$$I = \frac{100e^{100kt}}{99 + e^{100kt}}.$$

(b) If the constant of proportionality k has value 0.01 when t is measured in days, find the value of the rate of new cases $I'(t)$ at the end of each day for the first 9 days.

[1] In the English system where weight is measured in pounds, the unit of mass is the **slug** and is defined by slug = pounds/32.

(5) The supply of food for a certain population is subject to a seasonal change that affects the growth rate of the population. The differential equation

$$\frac{dx}{dt} = c\, x(t)\ \cos(t),$$

where c is a positive constant, provides a simple model for the seasonal growth of the population.

(a) Solve the differential equation in terms of an initial population x_0 and constant c.

(b) Determine the maximum and the minimum populations and the time interval between maxima.