SAMPLE APPLIED PROBLEMS
for
CHAPTER 2

(1) A body of mass 5 slugs \(^1\) dropped from a height of 100 feet with zero initial velocity. Assuming no air resistance, find

(a) an expression for the velocity of the body at time \(t\);
(b) an expression for the position of the body at any time \(t\);
(c) the time required for the body to reach the ground.

(2) A steel ball weighing 2 pounds is dropped from a height of 3000 feet with no initial velocity. As it falls, the ball encounters air resistance numerically equal to \((v/8)\) ft./pound-second. Find

(a) the limiting velocity of the ball in feet/second;
(b) the time required for the ball to hit the ground.

(3) A body of mass \(m\) is thrown vertically into the air with an initial velocity \(v_0\). If the body encounters air resistance proportional to its velocity, find

(a) the equation of motion;
(b) an expression for the velocity of the body at any time \(t\);
(c) the time at which the body reaches its maximum height.

(4) A college dormitory houses 100 students, each of whom is susceptible to a certain virus infection. A simple model of epidemics assumes that during the course of an epidemic the rate of change with respect to time of the number of infected students \(I\) is proportional to the number of infected students and is also proportional to the number of uninfected students.

(a) If at time \(t = 0\) a single student becomes infected, show that the number of infected students at time \(t\) is given by

\[
I = \frac{100e^{100kt}}{99 + e^{100kt}}.
\]

(b) If the constant of proportionality \(k\) has value 0.01 when \(t\) is measured in days, find the value of the rate of new cases \(I'(t)\) at the end of each day for the first 9 days.

\(^1\)In the English system where weight is measured in pounds, the unit of mass is the slug and is defined by \(\text{slug} = \text{pounds}/32\).
(5) The supply of food for a certain population is subject to a seasonal change that affects the growth rate of the population. The differential equation

\[ \frac{dx}{dt} = c \ x(t) \ \cos(t), \]

where \( c \) is a positive constant, provides a simple model for the seasonal growth of the population.

(a) Solve the differential equation in terms of an initial population \( x_0 \) and constant \( c \).

(b) Determine the maximum and the minimum populations and the time interval between maxima.