1. (25 points) Let \( \mathbf{u} = \hat{i} + 2\hat{j} - \hat{k} \), \( \mathbf{v} = -2\hat{i} - 4\hat{j} \), and \( \mathbf{w} = 7\hat{j} - 4\hat{k} \).

   (a) Find the projection of \( \mathbf{w} \) onto \( \mathbf{u} \times \mathbf{v} \).

   (b) Compute the volume of the parallelepiped determined by the vectors \( \mathbf{u}, \mathbf{v}, \) and \( \mathbf{w} \).

2. (25 points)

   (a) Find a unit normal to the plane containing the three points \( P : (1, 1, 7), Q : (3, 1, 5), \) and \( R : (2, 0, 3) \).

   (b) Write the equation of the plane of part (a) in the form \( (\mathbf{x} - \mathbf{p}) \cdot \mathbf{n} = 0 \). Rewrite this equation as a single equation involving the components \( x, y, \) and \( z \) of the vector \( \mathbf{x} \).

   (c) Find a vector perpendicular to the vector \( \mathbf{q} = (3, 1, 5) \) and which lies in the plane of part (a).

3. (25 points)

   (a) Find a parametric equation for the line passing through the point \( P : (1, 2, 3) \) and parallel to the vector \( \mathbf{v} = (-3, 0, 7) \).

   (b) Find the perpendicular bisector of the line segment joining the points \( P : (1, 2, 3) \) and \( Q : (4, 2, -4) \).

   (c) If \( \mathbf{m} \) is the position vector of the midpoint and \( \mathbf{r} \) is the position vector of the point \( (6, 2, 1) \), are the vectors \( \mathbf{m} - \mathbf{q} \) and \( \mathbf{v} \) linearly independent? Justify your answer.

4. (25 points)

   Let \( V \) be the vector space of points in \( \mathbb{R}^3 \).

   (a) If \( \mathbf{u}, \mathbf{v}, \) and \( \mathbf{w} \) are in \( V \), define the set \( \text{span} \{ \mathbf{u}, \mathbf{v}, \mathbf{w} \} \). What is its dimension?

   (b) For the specific choice \( \mathbf{u} = (2, -2, 1) \) and \( \mathbf{v} = (-1, 0, 1) \), identify \( \text{span}\{\mathbf{u}, \mathbf{v}\} \). (Hint: derive a single equation relating \( x, y, \) and \( z \) from a linear combination of \( \mathbf{u} \) and \( \mathbf{v} \).)