Math 114 - EXAM III 2001S SOLUTIONS

VERSION A (Blue/Green)

1. b
2. b
3. a
4. e
5. c
6. d
7. e
8. c
9. b
10. b
11. c
12. c
13. b
14. d

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Version A
Blue/Green

1. The graph is a "standard exponential function that is shifted down by 3 units. We know this because the usual y-intercept would be (0,1) and this graph has a y-intercept of $0,-2)$. Also, the horizontal asymptote is $y=-3$. Thus the equation would be $\boxed{y=5^x-3}$.

   The correct answer is **b**.

2. Simplify $\ln e^{(2x+5)}$.

   $\ln e^{(2x+5)} = (2x+5)*\ln e = (2x+5)*1 = \underline{2x+5.}$

   The correct answer is **b**.

3. Determine the value of log55. Use the Change of Base Formula.

   $$\log_5 55 = \frac{\log 55}{\log 5} = 2.4899$$

   The correct answer is **a**.

4. Rewrite as the logarithm of a single quantity.

   $2\log_3 x + \log_3(x+2)$

   Using Properties of Logs, $2\log_3 x + \log_3(x+2)$

   $$= \log_3 x^2 + \log_3(x+2)$$

   $$= \log_3 x^2(x+2)$$

   The correct answer is **e**.

5. Use the properties of logarithms to write the expression as a sum, difference, and/or multiple of logs.

$$\ln\left(\frac{2}{\sqrt{x^2+25}}\right) = \ln 2 - \ln\sqrt{x^2+25}$$

$$= \ln 2 - \ln(x^2+25)^{1/2}$$

$$= \ln 2 - \frac{1}{2}\ln(x^2+25)$$

The correct answer is **c**.

6. Give that $\log_b 3 = .6826$ and $\log_b 7 = 1.2091$, evaluate $\log_b 63$.

Using Properties of Logs:

$$\log_b 63 = \log_b(9*7) = \log_b(3^2*7)$$
$$= \log_b 3^2 + \log_b 7$$
$$= 2\log_b 3 + \log_b 7$$
$$= 2(0.6826) + 1.2091$$
$$= 1.3652 + 1.2091$$
$$= \underline{2.5743}$$

The correct answer is **d**.

7. Given $B = 10\log_{10}\left(\frac{I}{I_0}\right)$ where $I_0 = 10^{-16}$ watts/cm^2

and $I = 10^{-6.5}$ watts/cm^2, determine B.

$$B = 10\log_{10}\left(\frac{10^{-6.5}}{10^{-16}}\right)$$
$$= 10\log_{10}10^{9.5}$$
$$= 10*9.5*1$$
$$B = 95 \text{ decibels}$$

The correct answer is **e**.

8. Solve $\ln 2x - \ln 3 = 5$

$$\ln 2x - \ln 3 = 5$$
$$\ln\left(\frac{2x}{3}\right) = 5$$
$$e^5 = \frac{2x}{3}$$
$$\frac{2x}{3} = e^5$$
$$2x = 3e^5$$
$$x = \frac{3e^5}{2}$$

The correct answer is **c**.

9. Given that the fish population in a certain lake increases according to the equation

$$P = \frac{10,000}{1+15e^{\frac{-t}{5}}}$$ where t is measured in months.

Find P when t = 10 months.

$$P = \frac{10,000}{1+15e^{-2}} = 3300$$

The correct answer is **b**.

10. Solve for x:

$$9 - 4e^x = 6$$
$$-4e^x = -3$$
$$e^x = 0.75$$
$$x = \ln 0.75 \approx -.2877$$

thus, x is between -1 and 0.

The correct answer is **b**.

14. Given the data, determine the exponential regression equation which fits the data ($y = ab^x$ with "a" and "b" correct to 3 decimal places). Use this model to predict the population in the town in 2005.

t	1990 0	2	4	6	8	
Pop	2500	2700	2950	3450	4100	

Enter the data in two lists. Plot the data by accessing Stat Plot and designating the correct lists. Choose ZoomStat!

Follow this sequence to get the model.
Stat ⟶ CALC ⟶ 10. Exp Reg.

Exp Reg L_1, L_2 ⟵ (Designate Correct Lists)

The model is $y = 2413.888(1.064)^x$

Store the model in "$y =$" editor.
Graph the model.
Choose 2[nd] Trace ⟶ Value ⟶ $x = 15$.
(Since $0 = 1990$) [Will need to adjust window]
If $x = 15$, $y = 6121$

The correct answer is **d**.

11. Solve the following system of equations for the value of y:

$$6*\left(\frac{2}{3}x+\frac{1}{6}y=\frac{2}{3}\right) \longrightarrow 4x+y=4$$

$$5x-y=14 \longrightarrow + \; 5x-y=14$$

$$9x=18$$
$$x=2$$

Substitute $x=2$ in $5x-y=14$

$$10-y=14$$
$$y=-4$$

The correct answer is **c**.

12. If $8000 is deposited in an account that pays 6% interest compounded continuously for 10 years, how much will you have?

$A=Pe^{rt}$

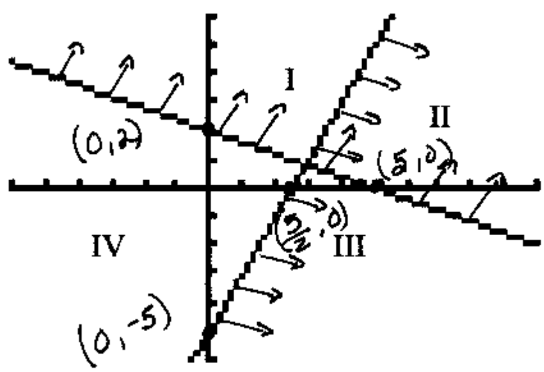
$A=8000e^{(.06*10)}=8000e^{0.6}=\$14,576.95$

The correct answer is **c**.

13. Determine which region defines the solution set for the system of inequalities.

	x-int.	y-int.	Test Value (0, 0)
$2x-y\geq 5$	$\left(\frac{5}{2}, 0\right)$	$(0,-5)$	$0\geq 5$ F
$2x+5y\geq 10$	$(5, 0)$	$(0, 2)$	$0\geq 10$ F

Since the arrows(shading) converge in Region II, it is the solution.



The correct answer is **b**.

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FREE RESPONSE

15. Given the following data set, generate in turn the linear, quadratic, and exponential models. [Correct to 4 decimal places]

a) Linear Model $y = .0104x + 1.3958$

$r = .0871$

b) Quadratic Model $y = -.0291x^2 + .4177x + .5813$

$R^2 = .9537$ or $r = .9766$

c) Exponential Model $y = 1.2205(1.0144)^x$

$r = .1510$

d) Explain which model best fits the data and why you made this decision.

The best model is the quadratic because this model best fits the shape of the data and passes through the most points. Also r-value is closest to 1.0.

e) Use your model to predict the number of Apple computers in the schools in 2001. [Round your answer to one significant digit.]

Use 2nd Trace ⟶ Value ⟶ $x = 15$.

$y = 0.3$ million computers.

16. The decay of radioactive radium (Ra^{226}) is given by

$$P = P_o e^{kt}$$

a) Use the fact that radioactive radium has a half-life of 1620 years to determine K. (5 decimal places). [4 pts]

$$e^{1620K} = .5$$

$$k = \frac{\ln .5}{1620} = -.42787 * 10^{-4}$$

$$K = -.00043$$

thus, $P = P_o e^{-.00043t}$

b) If 0.15 grams remain after 1000 years, determine the initial quantity of radioactive radium. (2 decimal places) [6 pts]

$$0.15 = P_o e^{(-.00043*1000)}$$

$$0.15 = P_o e^{-.43}$$

$$P_o = \frac{0.15}{e^{-.43}} = 0.23 \text{ grams}$$

Thus, 0.23 grams was the initial amount of radioactive radium.