1. Total number of passwords = $26 \times 26 \times 26 \times 26$.

   Total number of passwords with no $x$ or $y$ = $24 \times 24 \times 24 \times 24$.

   $$P\text{ (password with no } x \text{ or } y\text{)} = \frac{24\times24\times24\times24}{26\times26\times26\times26} = 0.73$$

2. $6 \times 38 = 228$

3. I. True
   II. True
   III. True

4. $\mu = 88,000$
   $\sigma = 12,500$

   Approximately 95% of all data fall within 2 standard deviations of the mean. $\mu \pm 2\sigma = 88,000 \pm 2(12,500) = 88,000 \pm 25,000$. The interval ranges from 63,000 cans to 113,000 cans.

5. 75% of the number of cans sold per week lie below $Q_3$.

   $Q_3 = \mu + .67\sigma$
   $Q_3 = 88,000 + .67\,(12,500)$
   $Q_3 = 96,375$

6. It is necessary to find the $z$-score which is associated with 87% of the area, or .8700. $\frac{1}{2}(.8700) = .4350$. From the body of the $z$-table, the $z$-score closest to an area of .4350 is 1.51.

   $88,000 \pm 1.51(12,500)$
   $88,000 - 1.51(12,500) = 69,125$
   $88,000 + 1.51(12,500) = 106,875$

   The interval ranges from 69,125 cans to 106,875 cans.

7. $\hat{p} = 67\%$

$$s_{\hat{p}} = \sqrt{\frac{67(100-67)}{900}}$$

Approximately 95% of all data fall within 2 standard deviations of the sample proportion.

$$\hat{p} \pm 2s_{\hat{p}} = 67\% \pm 2\sqrt{\frac{67(100-67)}{900}}$$

$$= 67\% \pm 3.13\%$$

8. \$3.60 is one-half of \$7.20. Therefore, the sample size would have to be increased by a factor of $2^2$ or 4. $4 \times 90 = 360$.

The new sample size would have to be 360.

9. I. True

II. False. Since $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$, the standard deviation of a distribution of sample means from a given population increases only if sample size decreases.

III. True.

10. A total of $43 + 107 = 150$ VCRs were purchased, given that brand $Q$ was purchased. Therefore, the percent of purchasers who bought a 4-head VCR = $\frac{43}{150} = 28.7\%$

11. A total of $57 + 95 = 152$ VCRs were purchased, given that brand M was purchased. Therefore, the percent of purchasers who bought a 2-head VCR = $\hat{p} = \frac{57}{152} = 37.5\%$.

$$s_{\hat{p}} = \sqrt{\frac{\hat{p}(100-\hat{p})}{n}} = \sqrt{\frac{37.5(100-37.5)}{152}} = 3.9\%$$

12. I. True.
II. False.
III. False.

**Name:** ______________________ **Section:** ________

**Instructor:** ______________________

**The following questions are free response. Please show all work in order to receive credit.**

13. Simultaneously, a spinner numbered 1 through 3 is spun and one die is rolled. The results are recorded. ( 10 pts. )

a. List all outcomes in the sample space.

(1, 1) (2, 1) (3, 1)
(1, 2) (2, 2) (3, 2)
(1, 3) (2, 3) (3, 3)
(1, 4) (2, 4) (3, 4)
(1, 5) (2, 5) (3, 5)
(1, 6) (2, 6) (3, 6)

b. Find the probability that at least one of the two numbers recorded is a 3.

P(at least one 3)

$= \frac{\text{number of outcomes with at least one 3}}{\text{total number of outcomes}}$

$= \frac{8}{18}$

c. Suppose the sum of the number spun and the number rolled is recorded. List all outcomes in this sample space.

$S = \{2, 3, 4, 5, 6, 7, 8, 9\}$

d. Find the probability that the sum recorded is at least 8.

$P(\text{sum recorded} \geq 8) = \frac{3}{18}$

14. Suppose a game has four outcomes: A, B, C, and D. The probability of outcomes A and D is 0.3; the probability of each of the remaining outcomes is 0.2. A player receives \$3 if outcome A occurs, \$4 if outcome B occurs, a \$2 if outcome C occurs, and must pay \$5 if outcome D occurs. (12 pts.)

a. Write the probability model for this game.

| | | | | |
|---|---|---|---|---|
| Outcomes | A | B | C | D |
| Winnings | \$3 | \$4 | \$2 | \$-5 |
| Probabilities | 0.3 | 0.2 | 0.2 | 0.3 |

b. Find the mean of the winnings.

$$\mu = s_1 p_1 + s_2 p_2 + s_3 p_3 + s_4 p_4$$
$$\mu = \$3(.3) + \$4(.2) + \$2(.2) + (\$-5)(.3)$$
$$\mu = \$.60$$

c. Find the standard deviation of the winnings.

$$\sigma^2 = (s_1 - \mu)^2 p_1 + (\ \ - \mu)^2 p_2 + (s_3 - \mu)^2 p_3 + (s_4 - \mu)^2 p_4$$
$$\sigma^2 = (3 - .6)^2(.3) + (4 - .6)^2(.2) + (2 - .6)^2(.2) + (-5 - .6)^2(.3)$$
$$\sigma^2 = 13.84$$
$$\sigma = \sqrt{13.84} = \$3.72$$

15. A sample of 16 fish is taken from a population and weighed. The mean weight is 5 pounds. The standard deviation of the weight of the fish population is 0.4 pounds. Find a 95% confidence interval for $\mu$, the population mean. ( 6 pts. )

$n = 16$

$\bar{X} = 5$

$\sigma = 0.4$

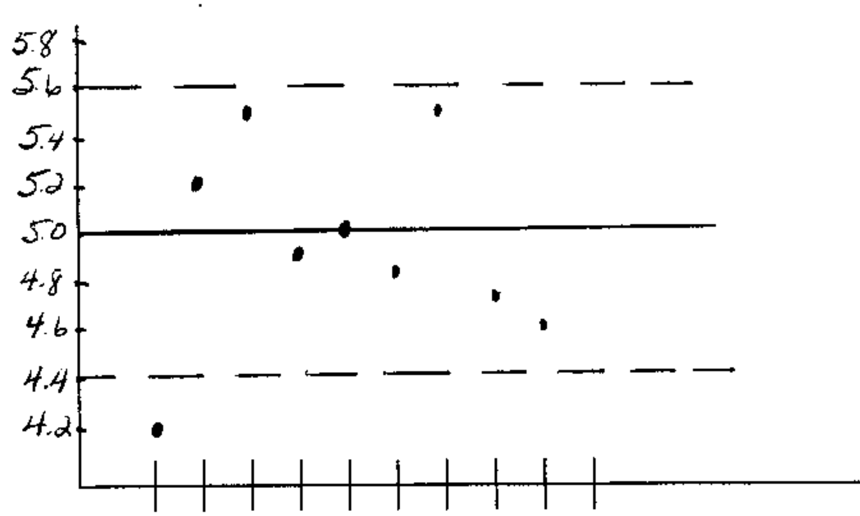
$$\bar{X} \pm 2\sigma_{\bar{X}} = 5 \pm 2 \cdot \frac{.4}{\sqrt{16}} = 5 \pm .2$$

Confidence Interval: $(4.8, 5.2)$

16. A company produces Chinese handcuffs, which are designed to be 5 inches in length. Every hour a sample of 16 is taken from the assembly line and the mean is found for the sample (see the table below). Assume that $\sigma = 0.8$ for the population of handcuffs. ( 12 pts.)

| Sample # | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $\bar{x}$ | 4.2" | 5.2" | 5.5" | 4.9" | 5" | 4.8" | 5.5" | 4.7" | 4.6" |

a. Make a statistical process control chart for this process.




Target value:
$\mu = 5$ lb

$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{0.8}{\sqrt{16}} = .2$

Control limits:
$5 \pm 3(.2) \Rightarrow$
4.4 and 5.6

b. Is this process out of control or in control? Explain your answer.

The process is out of control. A value of $\bar{X}$ is outside the control limits.