

- $6 \times 6 \times 6 = 216$
- The smallest sum is $3(1 + 1 + 1)$. The largest sum is $18(6 + 6 + 6)$. The total number of outcomes is 16 as there are 16 numbers between 3 and 18 inclusive.
- The sample space is $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$. Each outcome has probability $\frac{1}{8}$.
Three outcomes are in the event "exactly two heads". The probability of rolling exactly two heads is $3 \cdot \frac{1}{8} = \frac{3}{8}$.
- Total number of serial numbers is $5 \times 10 \times 9 \times 8$. Total number of serial numbers with no b or 7 is $4 \times 9 \times 8 \times 7$.

$$P(\text{serial number with no } b \text{ or } 7) = \frac{4 \times 9 \times 8 \times 7}{5 \times 10 \times 9 \times 8} = 0.5600$$

$$\begin{aligned} 5. \quad P(4 \text{ people}) &= 1 - (0.08 + 0.15 + 0.12 + 0.22 + 0.20) \\ &= 1 - 0.77 \\ &= 0.23 \end{aligned}$$

$$\begin{aligned} 6. \quad \mu &= 8.55 \text{ yr.} \\ \sigma &= 1.45 \text{ yr.} \\ n &= 10 \end{aligned}$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{10}} = \frac{1.45}{\sqrt{10}} = 0.459 \text{ yr}$$

7. In order to decrease the width of the confidence interval, decrease the level of confidence to less than 95%.
8. I. True
II. False. The mean of the normal distribution depends on the data itself.
III. False.
9. The age 16.1 yr is one standard deviation above the mean ($14.2 \text{ yr} + 1.9 \text{ yr} = 16.1 \text{ yr}$). Approximately 16% of the data lies more than one standard deviation above the mean.
10. Approximately 95% of the data fall within 2 standard deviations of the sample mean.

$$\begin{aligned}\bar{x} \pm 2\sigma_{\bar{x}} &= \bar{x} \pm 2 \cdot \frac{\sigma}{\sqrt{n}} \\ &= 15.1 \pm 2 \cdot \frac{1.9}{\sqrt{75}} \\ &= 15.1 \pm 0.4\end{aligned}$$

The 95% confidence interval is 14.7 yr to 15.5 yr.

Name: _____

Section: _____

The following questions are free response. Please show all work in order to receive credit.

11. (12 points)

- a. A parameter is a number that describes a population.
- b. If an outcome never occurs its probability is zero.
- c. According to the Central Limit Theorem when sample size is sufficiently large, the mean of the sampling distribution is equal to the population mean.
- d. Equally likely outcomes occur when every possible outcome of a random phenomenon has the same probability.
- e. According to the law of large numbers a random phenomenon is repeated many times, the mean \bar{x} of the observed outcomes approaches the mean μ of the probability model.
- f. A statistic is a number that describes a sample.

12. Suppose a game has four outcomes: A, B, C, and D. The probability of outcome A is 0.4; the probabilities of each of the other outcomes is 0.2. A player receives \$2 if outcome A occurs, \$3 if outcome B occurs, \$3 if outcome C occurs and must pay \$5 if outcome D occurs. (10 points)

a. Write the probability model for this game.

Outcomes:	A	B	C	D
	\$2	\$3	\$3	-\$5
probabilities	0.4	0.2	0.2	0.2

b. Find the mean of the winnings.

$$\mu = S_1 p_1 + S_2 p_2 + S_3 p_3 + S_4 p_4$$

$$\mu = 2(.4) + 3(.2) + 3(.2) + (-5)(.2)$$

$$\mu = \$1$$

c. Find the standard deviation of the winnings.

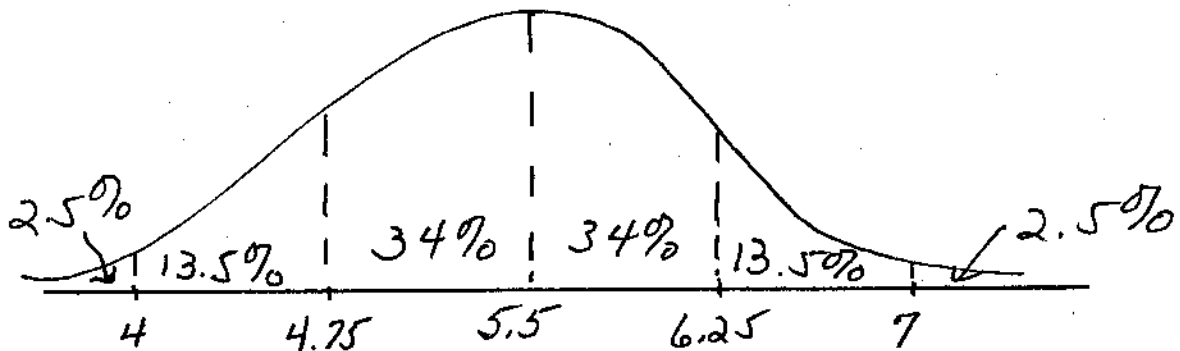
$$S^2 = (S_1 - \mu)^2 p_1 + (S_2 - \mu)^2 p_2 + (S_3 - \mu)^2 p_3 + (S_4 - \mu)^2 p_4$$

$$S^2 = (2 - 1)^2 (.4) + (3 - 1)^2 (.2) + (3 - 1)^2 (.2) + (-5 - 1)^2 (.2)$$

$$S^2 = 9.2$$

$$S = \sqrt{9.2} = \$3.03$$

13. The length of a student's college career at the University of Delaware is known to be normally distributed with a mean length of 5.5 years and a standard deviation of .75 yrs. (12 points)
- a. Sketch the normal curve. Locate the values of the mean and two standard deviations above and below the mean.



- b. What percent of students have college careers lasting between 4 and 7 years?

$$95\% = 13.5\% + 34\% + 34\% + 13.5\%$$

- c. Between what two years do the middle 50% of University of Delaware students fall?

$$Q_1 = \mu - 0.67\sigma$$

$$Q_3 = \mu + 0.67\sigma$$

$$Q_1 = 5.5 - 0.67(.75)$$

$$Q_3 = 5.5 + 0.67(.75)$$

$$Q_1 = 4.9975$$

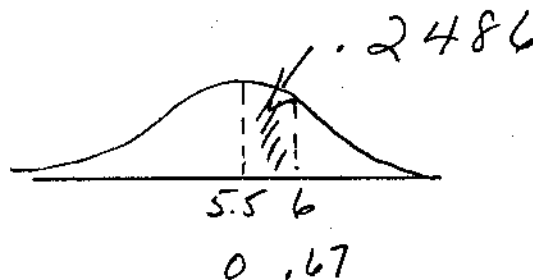
$$Q_3 = 6.0025$$

- d. Find the percentage of students who complete their college careers between 5.5 and 6 years (correct to 2 decimal places).

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{6 - 5.5}{.75}$$

$$Z = 0.67$$



24.86% will complete college between 5.5 & 6 yr.

14. A survey of 1500 voters finds that 1,100 voters support candidate Jones. (16 points)

a. Find the proportion of voters in the survey who support candidate Jones.

$$\hat{p} = \frac{1100}{1500} = 73.33\%$$

b. Find the standard deviation of the sampling distribution for this survey.

$$S_{\hat{p}} = \sqrt{\frac{\hat{p}(100-\hat{p})}{n}} = \sqrt{\frac{73.33(100-73.33)}{1500}}$$

$$S_{\hat{p}} = 1.14\%$$

c. Construct a 90% confidence interval for the proportion of voters who support candidate Jones.

$$90\% = .9000$$



$$\frac{.9000}{2} = .4500$$

From the z-table, $z = 1.65$

$$\begin{aligned}\hat{p} \pm z \cdot S_{\hat{p}} &= 73.33\% \pm 1.65(1.14\%) \\ &= 73.33\% \pm 1.88\%\end{aligned}$$

$$71.45\% \text{ to } 75.21\%$$

d. How large a sample is required to cut the standard deviation of the sampling distribution in half?

A sample of size 4,1500, or 6000, is needed to cut the standard deviation in half.