(xv_t, xv_{n-1})-minihypers in $\mathrm{PG}(t, q)$

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Let $v_i = \frac{q^i-1}{q-1}$. An (f, m)-minihypers in $\mathrm{PG}(t, q)$ is a weighted multiset $\mathfrak{F}$ of f points in $\mathrm{PG}(t, q)$ such that every hyperplane contains at least m points of $\mathfrak{F}$ (summing up multiplicities). In general, one can show that $f \geq \frac{v_t}{v_{t-1}} m$, hence (xv_t, xv_{t-1})-minihypers are the optimal minihypers with respect to their parameters. This is a first motivation to study these minihypers. The most common example of such a minihyper is the sum of any x hyperplanes, but several other examples are known [1, 5, 6].

A second motivation is that a Hamada showed a bijective correspondence between minihypers with certain parameter restrictions, and linear q-ary $[n, k, d]$ codes meeting the Griesmer [2, 7] bound

$$n \geq \sum_{i=0}^{k-1} \left\lceil \frac{d}{q^i} \right\rceil .$$

Out of all minihypers, the (xv_t, xv_{t-1})-minihypers are the ones giving the most highly divisible minimum distances.

We characterize the proper (xv_t, xv_{t-1})-minihypers in $\mathrm{PG}(t, q)$ as being a nonnegative *rational* sums of hyperplanes. This is a third motivation for the study of these objects, and more importantly, we can use this characterization to improve and extend several of the main results in previous papers on the subject [4, 5, 6]. Our rational result also allows new construction techniques, leading to several classes of (xv_t, xv_{t-1})-minihypers with previously unknown parameters.

References

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