Christoph Erath

National Center for Atmospheric Research, USA
University of Colorado at Boulder, USA
E-Mail: erath@ucar.edu
http://www.csc.cs.colorado.edu/~ce/

Abstract

Coupling of the Finite Volume Method and the Boundary Element Method

Numerical schemes, a priori and a posteriori results

We develop two discretization schemes for the coupling of the finite volume method and the boundary element method in two dimensions, which describe, for example, the transport of a concentration in a fluid. In a bounded interior domain we approximate a diffusion convection reaction problem either by the finite volume element method or by the cell-centered finite volume method, whereas in the corresponding exterior domain the Laplace problem is solved by the boundary element method. On the coupling boundary we have appropriate transmission conditions. Note that our model problem in the interior domain is the prototype for flow and transport of a concentration in porous media, which is often convection dominated. For this type of problem one might see the coupling approach as follows: instead of (the missing) Dirichlet and/or Neumann boundary conditions we assume a diffusion process in the corresponding (unbounded) exterior domain, which 'replaces' the boundary values. Both discrete systems maintain naturally local conservation of the numerical fluxes and a possible weighted upwind scheme guarantees the stability and flux conservation also for convection dominated problems.

We show existence and uniqueness of the continuous system. For the coupling with the finite volume element method we provide a convergence and an a priori analysis in an energy (semi-) norm, and an existence and uniqueness result, also for the upwind version. For both coupling systems we derive residual-based a posteriori estimates, which give upper and lower bounds for the error between the exact solution and the approximate solution. These bounds measure the error in an energy (semi-) norm and are robust in the sense that they do not depend on the variation of the model data. The local contributions of the a posteriori estimates are used to steer an adaptive mesh-refining algorithm. Numerical experiments show that our adaptive couplings are efficient methods for the numerical treatment of transmission problems, which exhibit local behavior and might be convection dominated in the interior domain.