Existence and Selection of Steady Needle Crystals

Abstract: This talk concerns the existence of solutions to a steady needle crystal growth problem in an one-sided model. We rigorously prove that for small nonzero anisotropy γ, analytic symmetric needle crystal solutions exist in the limit of surface tension ϵ if only if the stokes constant S for a relatively simple nonlinear differential equation is zero. This Stokes constant S depends on a parameter β which is related to crystalline anisotropy γ and surface tension ϵ. Earlier numerical calculations by a number of investigators have shown the Stokes constant S to be zero for a discrete set of values of β. It is also proven that for zero anisotropy, there can be no symmetric needle crystal solution in the considered space.

The methodology consists of two steps. First, the original problem is reduced to a weak half-strip problem for any γ in a compact set of $[0, 1)$ by relaxation of the symmetry condition. The weak problem is shown to have unique solution in the function space considered for any γ in any compact set of [0,1). When a symmetry is invoked, the weak problem is shown equivalent to the original needle crystal problem. Next, by considering the behavior of solution in neighborhood of an appropriate complex turning point for γ in a compact subset of [0,1), we extract an exponentially small term in ϵ as $\epsilon \to 0^+$ that generally violates the symmetric condition. We prove that the symmetry condition is satisfied for small ϵ when the parameter β is constrained appropriately.