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Model two dimensional Volterra type linear
Integral Equation with Fixed boundary
Singularity and super-singularity

Abstract

AbstractIn this lecture the two dimensional linear Volterra type of integral Equations containing singularity or super-singularity in the kernels function is considered. For different values of parameters is the integral equation existence theorems are proved for homogeneous and inhomogeneous equation.

Let $\mathcal{D}$ denote the rectangle

$$\mathcal{D} := \{(x,y) : a < x < a_0,\, b_0 < y < b\},$$

and

$$\Gamma_1 = a < x < a_0,\, y = b\,, \qquad \Gamma_2 = \{x = a, b_0 < y < b\}\,.$$

In the domain $\mathcal{D}$ we consider the following integral equation

$$u(x,y) + \lambda \int_a^x \frac{u(t,y)}{(t-a)^\alpha}\,dt - \mu \int_y^b \frac{u(x,s)}{(b-s)^\beta}\,ds + \delta \int_a^x \frac{u(t,y)}{(t-a)^\alpha}\,dt \int_y^b \frac{u(x,s)}{(b-s)^\beta}\,ds = f(x,y), \qquad (0.1)$$

where $\alpha > 0$, $\beta > 0$ and $\lambda, , \delta$ are constants and $f(x,y) \in C(\overline{\mathcal{D}})$ is a given function in $\overline{\mathcal{D}}$.

In this lecture the general solution of the Integral Equation 0.1constructed for the pairs $\alpha = 1, \beta = 1$. It will be established that for certain values of parameters and the corresponding homogeneous integral equation has an infinity number of linear independent solutions and for others values of λ, μ the corresponding homogeneous integral equation (0.1) has no other solutions than the trivial solution. Under some additional conditions it will be proved that the inhomogeneous integral equation (0.1) for certain values of λ, μ is solvable, and for some other values of λ, μ has a unique solution.