Explicit and Almost Explicit Spectral Calculations for Diffusion Operators

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The diffusion operator

$$H_D = -\frac{1}{2}\frac{d}{dx}a\frac{d}{dx} - b\frac{d}{dx} = -\frac{1}{2}\exp(-2B)\frac{d}{dx}a\exp(2B)\frac{d}{dx},$$

where $B(x) = \int_0^x \frac{b}{a}(y)dy$, defined either on $R^+ = (0,\infty)$ with the Dirichlet boundary condition at $x = 0$, or on R, can be realized as a self-adjoint operator with respect to the density $\exp(2Q(x))dx$. The operator is unitarily equivalent to the Schrödinger-type operator $H_S = -\frac{1}{2}\frac{d}{dx}a\frac{d}{dx} + V_{b,a}$, where $V_{b,a} = \frac{1}{2}(\frac{b^2}{a} + b')$. We obtain an explicit criterion for the existence of a compact resolvent and explicit formulas up to the multiplicative constant 4 for the infimum of the spectrum and for the infimum of the essential spectrum for these operators. We also compare our results to some known results on weighted Hardy inequalities and give applications to self-adjoint, multi-dimensional diffusion operators. The methods and statements of the results are analytic, but the intuition and some of the motivation comes from probabilistic considerations.