OPTIMAL ANISOTROPIC ERROR ESTIMATES AND APPLICATIONS TO CONVECTION DOMINATED PROBLEMS

LONG CHEN

In this talk, we first present an interpolation error estimate in L^p norm $(1 \leq p \leq \infty)$ for finite element simplicial meshes in any spatial dimensions and then discuss its applications to convection dominated problems. We show that an asymptotically optimal error estimate can be obtained under near optimal meshes. A sufficient condition for a mesh to be nearly optimal is that it is quasi-uniform under a new metric defined by a modified Hessian matrix of the function to be interpolated. We further show such estimates are in fact asymptotically sharp for strictly convex functions. To illustrate the useful of optimal meshes, we give an exact gradient recovery formula and briefly discuss some interesting and related problems in the computational geometry, such as sphere covering and mesh smoothing.

The above interpolation error estimate is useful for approximating functions with anisotropic singularity. Thus it can be applied to convection diffusion problem with small diffusion parameter ϵ, of which solutions usually present boundary layers or interior layers. We are interested in when and how discretization errors may be governed by interpolation errors. We show, theoretically and numerically, that the discretization error of the standard FEM is sensitive to the perturbation of the grid points in the region where the solution is smooth. We have carefully designed a special streamline diffusion finite element method whose discretization error is shown to be uniformly governed by the interpolation error in maximum norm. For problems in multidimensions, we shall discuss some practical issues in the algorithms especially the homotopy with respect to the parameter ϵ. Our overarching goal is to develop and analyze discretization schemes for one-parameter family PDEs, which is stable and accurate uniformly with respect to the parameter.