Geometric analysis on some subRiemannian manifolds

We propose a possible structure for geometrically invariant formulas for the fundamental solutions and heat kernels of partial differential operators of the form

$$\Delta_{\mathbf{X}} = \frac{1}{2}\left(X_1^2 + \cdots + X_m^2\right),$$

where $X_1, \ldots, X_m$ are m linearly independent vector fields on $\mathcal{M}_n$, an n-dimensional real manifold with $m \leq n$. More precisely, we give explicit description of the geometry induced by the operator $\Delta_{\mathbf{X}}$. Using Hamilton-Jacobi theory, we obtain the number of geodesics connecting any two points on the manifold. We also construct a complex action function whose critical points give lengths of those geodesics. Then we construct the fundamental solution for $\Delta_{\mathbf{X}}$ in terms of this action and volume element.