

Symmetry Analysis of PDEs: Classical, Nonclassical and Beyond

Abstract: Symmetry analysis has played an important role in the construction of exact solutions to nonlinear partial differential equations. This is based on the original work of Lie (1881) on continuous groups and provides a unified explanation for the seemingly diverse and ad-hoc integration methods still used to solve ordinary differential equations.

In this talk, both classical and nonclassical methods will be discussed with a focus on nonlinear reaction-diffusions equations. It will be shown that for several classes of one-dimensional nonlinear diffusion and reaction, the equations admit nontrivial symmetries

Recent results indicate that these symmetries can be obtained through compatibility with lower order equations. The heat equation serves to highlight this for quasi-linear evolution equations while the Boussinesq equation, a 2+1 dimensional nonlinear diffusion equation and a nonlinear wave equation system serve to show that this is true in general.

Results will be presented illustrating the connection between compatibility (sometimes referred to as Charpit's method) of first order PDEs and nonclassical contact symmetries. Generalized compatibility for a class of 2+1 nonlinear diffusion equation with a source term will also be considered suggesting possible future directions.