Convergence Analysis of Adaptive Finite Element Methods for Constrained Distributed and Boundary Control Problems

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Abstract

In this contribution, we are concerned with the development, analysis and implementation of adaptive finite element methods for distributed and boundary control problems with control constraints. The methods presented in this contribution provide an error reduction and thus guarantee convergence of the adaptive loop which consists of the essential steps 'SOLVE', 'ESTIMATE', 'MARK', and 'REFINE'. Here, 'SOLVE' stands for the efficient solution of the finite element discretized problems. The following step 'ESTIMATE' is devoted to a residual-type a posteriori error estimation of the global discretization errors in the state, the co-state, the control and the co-control. A bulk criterion is the core of the step 'MARK' to indicate selected edges and elements for refinement, whereas the final step 'REFINE' deals with the technical realization of the refinement process itself.
The analysis is carried out for a model problem using discretizations of the state and the co-state by continuous, piecewise linear finite elements and of the control and the co-control by element-wise constants with respect to a simplicial triangulation of the computational domain. Important tools in the convergence proof are the reliability of the estimator, a discrete local efficiency, and a perturbed Galerkin orthogonality.
Numerical results illustrate the performance of the error estimator.

References

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