Syllabus for the Probability (M630, M631, M850)
Candidacy Exam
Approved by the graduate committee on September 14, 2005

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• Probability space and measure; sigma algebras; Caratheodory's extension theorem; Kolmogorov's extension theorem; independence.

• Random variables; expected value; joint distributions; moment generating functions; characteristic functions; Chebyshev's inequality; large deviations.

• Modes of convergence (weak, in probability, L^p, almost sure) and properties.

• Weak laws of large numbers for sequences and triangular arrays; L^2 weak law; applications to analysis and combinatorics.

• Borel-Cantelli lemmas; strong law of large numbers; convergence of random series.

• Weak convergence; central limit theorem for sequences and triangular arrays; Poisson convergence; applications.

• Stopping times; Wald's equation; conditional probability/expectation; Martingales; Radon-Nikodym theorem; optional stopping theorems; applications.

• Stochastic processes and their properties; discrete and continuous time Markov chains; birth-death process and Poisson process; renewal theory.

• Brownian motion and related Gaussian/diffusion processes; Donsker's theorem.

• Stochastic simulations.

References:

1. *Introduction to Probability Models* by Sheldon Ross (Eighth edition): Chapters 1-7, 10-11.

2. *Probability and random processes*, by Geoffrey R Grimmett and David R. Stirzaker (Third edition): Chapter 1-10, 12-13.

3. *Probability and random processes, problems and solutions*, by Geoffrey R Grimmett and David R. Stirzaker.

4. *Probability: Theory and Examples*, by Richard Durrett (Third edition): Chapters 1-4, 7.