Monotonic Sequence Games

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A permutation $\pi = a_1 a_2 \ldots a_n$ in the symmetric group S_n has an *increasing subsequence of length* m if there are indices $i_1 < i_2 < \ldots < i_m$ such that $a_{i_1} < a_{i_2} < \ldots < a_{i_m}$. Decreasing subsequences are defined similarly. A famous theorem of Erdős and Szekeres states that any permutation in S_{mn+1} has either an increasing subsequence of length $m+1$ or a decreasing subsequence of length $n+1$. In a monotonic sequence game, two players form a sequence by alternately choosing distinct elements from the set $\{1, 2, \ldots, mn+1\}$. The game ends when the resulting sequence contains either an increasing subsequence of length $m+1$ or a decreasing one of length $n+1$. By the Erdős-Szekeres Theorem, this must happen at some point. Harary, Sagan, and West were the first ones to investigate this game and determine who wins when either m or n is sufficiently small. We investigate the behaviour of this game when played by choosing elements from other partially ordered sets. In particular, using rational numbers yields interesting results via a variant of the Robinson-Schensted-Knuth correspondence.