UNIVERSITY OF DELAWARE
DEPARTMENT OF MATHEMATICAL SCIENCES
DISCRETE MATHEMATICS SEMINAR

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A generalization of Sylow's theorems on finite groups to association schemes

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Let (X, G) denote an association scheme and let p denote a prime number. An element g in G will be called p-valenced if n_g is a power of p. A subset of G will be called p-valenced if each of its elements is p-valenced. A p-valenced subset F of G will be called a p-subset if n_F is a power of p.

Let us assume (X, G) to be finite. Then, by definition, n_G is finite. We shall denote by $\mathrm{Syl}_p(G)$ the set of all closed p-subsets H of G such that p does not divide $n_{G//H}$.

Theorem. *Let (X, G) denote a finite scheme, let p denote a prime number, and let P denote a closed p-subset of G. Then, if G is p-valenced, we have the following.*

1. *If $P \notin \mathrm{Syl}_p(G)$, there exists a closed p-subset P of G such that $P \subseteq P' \subseteq N_G(P)$ and $pn_P = n_{P'}$.*

2. *For each element P in $\mathrm{Syl}_p(G)$, there exists an element g in G such that $gPg^* \subseteq P'$. (If $P \in \mathrm{Syl}_p(G)$, $gPg^* = P'$.)*

3. *Assume that $P \in \mathrm{Syl}_p(G)$, and set $N := N_G(P)$. Then we have $n_{G//N} \equiv 1(p)$ and $n_{G//N} \equiv |\{gPg^* \mid g \in G\}) \cap \mathrm{Syl}_p(G)|(p)$.*