UNIVERSITY OF DELAWARE
DEPARTMENT OF MATHEMATICAL SCIENCES
DISCRETE MATHEMATICS SEMINAR

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Proper finite loop of order 6 having a regular group of collineations: A positive answer on a question of Barlotti-Strambach

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Recall that when a Latin square Q defines a group, the automorphism group $G = \mathrm{Aut}(L(Q))$ of the Latin square graph $L(Q)$ is transitive and contains a regular subgroup isomorphic to Q^2. For this reason one may say that a quasigroup defined by Q is, in a sense, "close to a group" provided G is a transitive permutation group of degree n^2, where n is the order of Q. Investigation of various classes of such quasigroups Q was conducted by a number of authors. Here we consider an additional property of G which arises from a question posed by A. Barlotti and K. Strambach in 1988: Does there exist such quasigroup Q which does not contain a group in its main class and for which G has a regular subgroup?

We provide first such example which was originally obtained with the aid of a computer. Surprisingly, it is one of the well known Latin squares of order 6; its group has order 648. We consider a computer-free, ad hoc model of $L(Q)$ (and also of G), and we outline a way in which this example can be generalized. Various interesting historical facts about the investigations of small latin squares will be naturally included and discussed in the course of our presentation.

This talk is based on a collaboration with A. Heinze, A. Rosa, A. Woldar and other mathematicians.