

UNIVERSITY OF DELAWARE
DEPARTMENT OF MATHEMATICAL SCIENCES
DISCRETE MATHEMATICS SEMINAR

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Co-degree density of hypergraphs

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Given a family F of r -uniform hypergraphs, the classical Turán theory studies the maximum proportion of r -sets an n element set can have without containing any member of F . This limit, as n becomes large, is sometimes called the Turán density of F . We consider the related problem of determining the (normalized) maximum possible minimum co-degree that the family of r -sets in an n element set can have without containing any member of F . As n goes to infinity, this approaches a limit which we call the co-degree density of F , and write $\gamma(F)$.

For each $r > 1$, let $G_r = \{\gamma(F) : F \text{ is a family of } r\text{-graphs}\}$. Our main result is that for each $r > 2$, G_r is dense in $[0, 1)$. This is in stark contrast to the fact that $G_2 = \{0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4} \dots\}$, a fact that follows from the Erdős-Simonovits-Stone theorem. This phenomenon is similar to the existence of real numbers that are not jumps in hypergraphs, proved by Frankl and Rödl. Several other results about co-degree densities are provided parallel to those of the classical Turán theory.

This is a joint work with Dhruv Mubayi.