

UNIVERSITY OF DELAWARE
DEPARTMENT OF MATHEMATICAL SCIENCES
DISCRETE MATHEMATICS SEMINAR

Friday Dec. 5, 2003, 4:00pm, Room 436 Ewing Hall

The exact Turán number of the Fano plane

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The Fano plane is the unique hypergraph with 7 triples on 7 vertices in which each pair of vertices is contained in a unique triple. Its edges correspond to the lines of the projective plane over the field with two elements. The Turán problem is to find the maximum number of edges in a 3-uniform hypergraph on n vertices not containing a Fano plane.

Noting that the Fano plane is not 2-colourable, but becomes so if one deletes an edge, a natural candidate is the largest 2-colourable 3-uniform hypergraph on n vertices. This is obtained by partitioning the vertices into two parts, of sizes differing by at most one, and taking all the triples which intersect both of them. Denote this hypergraph by $H_2(n)$.

We show that for sufficiently large n , the unique largest 3-uniform hypergraph on n vertices not containing a Fano plane is $H_2(n)$, thus proving a conjecture of V Sós raised in 1976.

This is joint work with Benny Sudakov.